Monopolisation and Industry Structure

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Abstract

The aim of this paper is to provide empirically testable predictions regarding the relationship between market size and concentration. In a model of endogenous horizontal mergers, it is shown that *concentrated outcomes* cannot be supported in a free entry equilibrium in large exogenous sunk cost industries: the *upper bound to concentration* tends to zero as market size (relative to setup costs) tends to infinity. In contrast, arbitrarily concentrated outcomes may be sustained in endogenous sunk cost industries, no matter how large the market, and even in the absence of mergers; that is, the upper bound to concentration does not decrease with market size. Using an equilibrium concept defined in the space of observable outcomes, it is shown that these predictions do not depend on the details of the extensive form of the game, even allowing for side payments between firms and endogenous product choice. The results complement those of Sutton (1991) on the stability of *fragmented outcomes*.

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1 Introduction

For a long time, a major topic in the literature on industrial market structure has been to explain differences in concentration across industries by referring to a small number of explanatory variables. Since the introduction of formal game-theoretic modelling into industrial organisation (I.O.), however, cross-industry studies have become less popular. The dilemma of the game-theoretic approach to I.O. appears to be twofold. First, equilibrium outcomes often depend delicately on features of the model that are both unobservable to an empirical researcher and likely to vary from one industry to another. Second, even if we can pin down the specification of the game, the problem remains that many models have multiple equilibria. One response to this dilemma is the "bounds approach to concentration", developed by Sutton (1991). The idea underlying this approach is to divide the space of outcomes into those that can be sustained as equilibrium outcomes in a broad class of admissible models and those that cannot.

Sutton (1991) applied the bounds approach to study the relationship between concentration and market size. He showed that the alleged negative relationship between market size and concentration breaks down in certain groups of industries. In particular, he introduced the distinction between "exogenous" and "endogenous" sunk cost industries. In the former, the only sunk costs involved are the exogenously given setup costs; R&D and advertising outlays are insignificant. In the latter, however, the equilibrium level of sunk costs is endogenously determined by firms' investments decisions. Roughly, these are industries where advertising or R&D are effective in that investments in some fixed outlays raise consumers' willingness-to-pay, or reduce marginal costs of production. Sutton predicted that, in exogenous sunk cost industries, the lower bound to concentration (i.e. the lower bound to the set of outcomes sustainable in equilibrium) tends to zero as the market becomes large, whereas in endogenous sunk cost industries, the lower bound to concentration is bounded away from zero, no matter how large the market. That is, in endogenous sunk cost industries, very fragmented outcomes (in the sense of low one-firm concentration ratios) cannot be supported as equilibrium outcomes in large markets; such outcomes cannot be excluded in exogenous sunk cost industries.[1]

Although robust, Sutton's predictions may not be entirely satisfactory in that they

[1]The "nonfragmentation" or "nonconvergence" result for endogenous sunk cost models has been formally shown by Shaked and Sutton (1987). The predictions have been empirically tested in Sutton (1991) and Robinson and Chiang (1996), amongst others.

are quite "weak": they refer to the stability of fragmented outcomes in large markets only. An important open question, raised by Bresnahan (1992) and others, is whether or not it is possible to make tighter predictions regarding the size-structure relationship.

The aim of this paper is to investigate the kinds of industries in which arbitrarily *concentrated* outcomes can and cannot be sustained. In other words: is there a nontrivial *upper bound to concentration*? There appear to be limits to monopolisation of industries, as evidenced by inspection of Sutton's dataset. In particular, in the exogenous sunk cost case, the data points do not "fill" the space above the lower bound; see Figure 1.

What are the trade-offs firms may face in their attempts to monopolise markets? The economic history of the U.S. around 1900 provides many examples of "attempts of monopolisation". At a time when mergers were not yet scrutinised by antitrust authorities, firms tried to monopolise industries by horizontal mergers. The most famous case is probably the formation of the United States Steel Corporation through consolidation of twelve steel producers in 1901. In their study of the U.S. Steel case, Parsons and Ray (1975) argue that it was primarily motivated by a successful attempt to gain market power, and *not* by efficiency considerations. An important feature of this case (as well as of many other merger cases) was the steady decline in U.S. Steel's market share. For instance, its market share of steel ingot and casting production decreased from 65.4% in 1902 to 54% in 1911, and 38.9% in 1929.[2] According to Stigler (1950) , the observed decline in market share was due to new entry and the expansion of fringe firms.[3] The lesson to be learned from the U.S. Steel merger case may be summarised as follows. Firms have an incentive to monopolise the market in order to gain market power. In the absence of antitrust authorities, this can be accomplished by horizontal mergers. However, two mechanisms make it difficult to persistently monopolise an industry: *i)* It may not be possible to persuade all firms in the industry to merge, since some firms may be better off not merging, given that rival firms still have an incentive to merge. *ii)* Even if all firms in the industry decided to merge, this may trigger new entry. Indeed, the weaker the competition in the market, the more profitable entry becomes. The existing game-theoretic literature on endogenous horizontal mergers (e.g. Kamien and Zang,

[2]This reduction in market share was abated by U.S. Steel's aggressive purchase of ore deposits. The corresponding rise in market price for ore sharply reduced the profitability of entry.

[3]Parsons and Ray (1975) provide a revealing example of free-riding behaviour of fringe firms. Whereas National Steel's share of total ingot capacity was just 2.5% in 1930, its ingot capacity expansion accounted for 25% of total steel ingot expansion during the 1930s. There were many years in which the capacity utilisation rate of its steel production subsidiary was about double the industry average.

1990) has focused mainly on the first mechanism. Here, we follow this tradition in the first part of the paper (Section 2). In the second part of the paper (Section 3), we then explore the second mechanism.

In the first part of the paper, we study the limits of monopolisation by investigating the market structure that would emerge if firms were free to merge in the absence of any antitrust laws. As pointed out by Stigler (1950), the resulting market structure is not necessarily a monopoly since firms face a trade-off between participation in a merger (so as to achieve a less competitive outcome) and nonparticipation (so as to free ride on the merging firms' efforts to restrict output). We consider an industry with horizontally differentiated products and non-localised competition. Upon entry, each firm is equipped with the know-how to produce one distinct product; the product portfolio of post-merger coalitions is the collection of products offered by its members.

In this paper, we distinguish between exogenous and endogenous sunk cost industries. In the exogenous sunk cost case, the endogenous horizontal merger game consists of three stages: *i)* firms decide whether or not to enter the industry and those which decide to enter have to pay some entry fee; *ii)* the firms that have decided to enter form "coalitions"; and *iii)* the newly formed coalitions compete in prices. We show that, for a given number of firms in the industry, merger to monopoly obtains as long as products are sufficiently good substitutes. For a given degree of substitutability of products, however, monopoly does not emerge endogenously if the number of entrants is sufficiently large. Moreover, in any equilibrium, the market share of the largest coalition converges to zero as the number of entrants tends to infinity. Now, in a free entry equilibrium, an increase in the size of the market raises the profitability of entry and hence the number of entering firms. In the limit as market size goes to infinity, the number of firms increases without bound. This implies that concentrated outcomes cannot be sustained in large markets. *In exogenous sunk cost industries, the upper bound to concentration goes to zero as market size tends to infinity.*

To analyse the endogenous sunk cost case, we allow firms to invest in fixed outlays so as to improve the quality of their products. In this case, the model comprises an additional investment stage. We show that – even in the absence of mergers – the equilibrium number of entering firms remains finite, no matter how large the market. This is the nonfragmentation result for endogenous sunk cost industries. Moreover, if products are sufficiently good substitutes (or investment in quality is sufficiently effective), then only one firm can be supported in equilibrium. Thus, not even in large markets is it in general possible to exclude arbitrarily concentrated outcomes. That is, *in endogenous sunk cost industries, the upper bound to concentration does not decrease with the size of the market.*

In the second part of the paper, we investigate the extent to which "ex-post entry" constrains the emergence of concentrated outcomes. The key feature of the assumed extensive form of the game is that there is some penultimate stage (before competition takes place) at which new firms can enter the market. This formalises Stigler's notion of "post-merger entry". The important insight is that the details of the extensive form do not matter, once we allow for this key feature. In particular, our structure is flexible enough to allow for a large class of noncooperative merger (or coalition formation) games. Moreover, firms may or may not make side payments and monopolise markets not only through mergers but also through product proliferation. We use the *One Smart Agent* equilibrium concept (Sutton, 1997). It is defined not in the space of strategies but in the space of (observable) outcomes. This noncooperative equilibrium concept involves two rather weak assumptions, both of which are implied by subgame perfection. Reassuringly, our main predictions coincide with those of the first part of the paper, where we do not allow for ex-post entry and put more structure on the extensive form. In exogenous sunk cost industries, the upper bound to concentration does indeed go to zero as market size (relative to setup costs) tends to infinity. In contrast, monopoly outcomes may be sustained in endogenous sunk cost industries, no matter how large the market. Hence, *allowing for ex-post entry, the predictions of this paper hold independently of any details regarding coalition formation or product selection.*

How are our predictions borne out by the data? Figures 1 and 2 reproduce Sutton's (1991) data set on the food and drink sector. The scattered points in the (concentration, market size)-space are at least suggestive of the predicted relationships between the upper bound to concentration and market size (relative to setup costs) in exogenous and endogenous sunk cost industries, respectively. For exogenous sunk cost industries, the upper bound decreases sharply with market size, while it stays flat in the case of endogenous sunk cost industries.

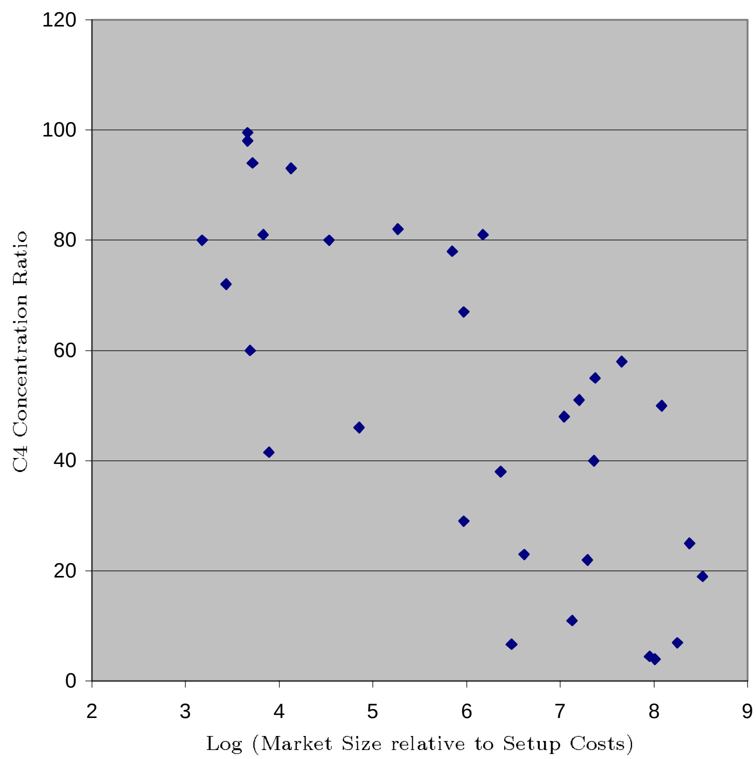

Figure 1: Exogenous Sunk Cost Industries. Data Source: Sutton (1991).

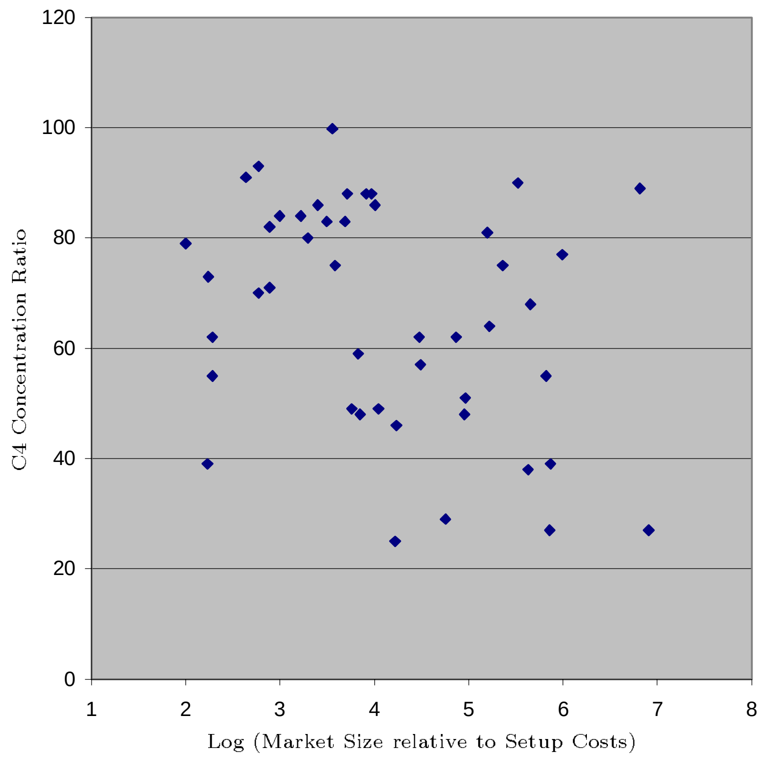

Figure 2: Endogenous Sunk Cost Industries. Data Source: Sutton (1991).

2 Endogenous Horizontal Mergers and the Upper Bound to Concentration

Using a model of endogenous horizontal mergers, we now investigate whether concentrated outcomes can be sustained as equilibrium outcomes in large markets. Coalition formation is modelled as a noncooperative open membership game; post-merger entry is not considered here. As explained earlier, we distinguish between exogenous and endogenous sunk cost industries.

2.1 Monopolisation in Exogenous Sunk Cost Industries

We begin by analysing the limits of monopolisation in exogenous sunk cost industries, where R&D and advertising outlays are insignificant; the only kind of sunk costs involved are exogenously given by firms' setup costs.

2.1.1 The Model

We consider an industry offering a potentially infinite number of substitute goods to S identical consumers. A consumer's utility is given by

$$U(\mathbf{x}; H) = \sum_{k=1}^{\infty} \left(x_k - x_k^2\right) - 2\sigma \sum_{k=1}^{\infty} \sum_{l<k} x_k x_l + H, \tag{1}$$

where x_k is consumption of substitute good k, and H is consumption of the Hicksian composite commodity whose price is normalised to one. Let Y denote income and p_k the price of good k. Then, $H = Y - \sum_k p_k x_k$. The quadratic utility function (and the associated linear demand system) goes back to Bewley (1924).[4] It is widely used in oligopoly models; see Vives (1999).[5] It defines utility over the domain of $\mathbf{x}$ for which all the marginal utilities $\partial U(\mathbf{x}; H)/\partial x_k$ are nonnegative. The form of the utility function ensures that a consumer's inverse demand for any good is linear. Income Y is assumed to be sufficiently large, $Y > 1/8\sigma$, so that $Y > \sum_k p_k x_k$ in equilibrium. The parameter σ, $\sigma \in (0, 1)$, measures the degree of substitutability between products. In the limit as $\sigma \to 1$, goods become perfect substitutes; in the limit as $\sigma \to 0$, products become

[4] I am grateful to Stephen Martin for providing this reference.

[5] For instance, Shubik and Levitan (1980), Deneckere and Davidson (1985), and Shaked and Sutton (1990) all consider price competition, whereas Sutton (1997, 1998) analyses quantity competition using these preferences.

independent. All goods (other than the outside good) are treated symmetrically. The choice of the demand system is for technical convenience: its symmetry allows us to consider general coalition structures.

We consider the following three-stage game. There are n_0 potential entrants, each of which has the know-how to produce a unique substitute good. At the first stage, these n_0 firms decide (either simultaneously or sequentially) whether or not to enter the industry. Entry costs in the industry are given by ϵ, $\epsilon > 0$. Since we confine our attention to free-entry equilibria, we assume that n_0 is sufficiently large, $n_0 > \left[S - 8\epsilon(1-\sigma)\right]/(8\epsilon\sigma)$, so that in any equilibrium of the game there is at least one nonentering firm. At the second stage, the firms that have decided to enter at the preceding stage simultaneously decide which "coalition" to join. All firms that have decided to join the same coalition then merge. Formally, firm k selects $z_k = i$, $i \in \mathbb{Z}$, if it decides to join coalition M_i. A coalition structure $\left\{(M_i)_{i\in\mathbb{Z}}\right\}$ is thus an endogenous partition of the set of entrants, induced by the vector of participation decisions, $\mathbf{z}$. Since firms can join an infinite number of coalitions (almost all of which will be empty in equilibrium), arbitrary coalition structures are allowed to emerge in equilibrium. This coalition formation game is sometimes called an open membership game; see Yi (1997).[6] At the third and final stage, the newly formed coalitions, each offering the products of its "members", compete simultaneously in prices. Each coalition faces a constant marginal cost of production, c; to simplify notation, we assume $c = 0$.

Each of the n_0 firms is assumed to act so as to maximise its profit; the same applies to the merged entities at the price-setting stage of the game. The members of each coalition share the coalition's profit. Since firms are symmetric, we assume, for simplicity, that profit is shared equally. The "equal sharing rule" can be obtained from various m-player bargaining solutions.

2.1.2 Equilibrium Analysis

We now seek the pure strategy subgame perfect equilibrium (SPE) of the three-stage game.

Price-Setting Stage. We solve for the SPE of the game by backward induction, starting with the third stage where the merged entities simultaneously compete in prices. Suppose that n firms have entered the industry at the first stage, relabel them as firms

[6] Mergers in this game may be thought of as "all share mergers": the owners of the merging firms obtain shares in the new company instead of cash.

1 to n, and let $N \equiv \{1, ..., n\}$. We take firms' merger decisions, described by the vector $\mathbf{z} = (z_1, ..., z_n)$, as given. Let m_i denote the number of members of coalition M_i, i.e. $m_i \equiv \#\{k \mid z_k = i\}$. The following lemma summarises equilibrium behaviour at the price-setting stage.

Lemma 1 *For any vector of merger decisions* $\mathbf{z}$*, there exists a unique Nash equilibrium in prices. In equilibrium, each coalition sets the same price for all of its products and each coalition makes a positive profit.*

Proof. See Apendix A. ■

In this context, several comparative statics results can be obtained regarding price. First, for a given coalition structure, the equilibrium price is strictly increasing in the size of the coalition. That is, an increase in the price of a product exerts a positive externality on the demand for all other products. Each coalition internalises the externality of a price change on its own products; the greater the number of products in a merged entity, the higher the price of its products. Second, for a given number of own products, the coalition's price is strictly increasing in industry concentration in the following sense. Consider any two coalitions, say, M_j and M_l, where $m_j \geq m_l$. Then, any increase in $m_j - m_l$ that leaves $m_j + m_l$ unchanged, raises the equilibrium price of coalition M_i, $i \neq j, l$.[7] The intuition is that an increase in the average price of rival goods, due to an increase in concentration, induces a coalition to raise its own price as well since prices are strategic complements. Third, if $\sigma \to 1$, then, for all coalition structures other than monopoly, each coalition's price converges to the competitive price, which is equal to zero; this limit case is the famous "Bertrand paradox". If $\sigma \to 0$, then goods become independent, and the price converges to the monopoly price of 1/2.

As regards equilibrium profit, we observe that *i)* a coalition's profit per product is decreasing in the number of its products, holding the coalition structure fixed, and *ii)* a coalition's profit per product is increasing in industry concentration for a given size of the coalition. The argument for the first observation is twofold. Consider the price-setting of a given product, fixing the prices of the $n-1$ other products at their equilibrium values. According to the first part of the argument, a one-product coalition sets price so as to maximise the profit of this product, whereas a multiproduct coalition sets a higher price since this imposes a positive externality on the coalition's other products. Clearly, the

[7] To see this, consider equation (16) in Appendix A and observe that the function $\xi(m) \equiv m/[2(1-\sigma) + (2n-m)\sigma]$ is strictly convex.

larger a coalition's product portfolio, the larger the upward bias in pricing. The second part of the argument consists in noting that the larger the coalition owning the product under consideration, the lower the average equilibrium price of the $n-1$ other products. This follows from the fact that price is increasing in coalition size.

Merger Stage. In the second stage of the game, a vector of merger decisions, $\mathbf{z}^*$, can be sustained in an SPE of the subgame in which n firms enter if and only if

$$\pi^*_{M_{z_k^*}}(z_k^*, \mathbf{z}^*_{-k}) \geq \pi^*_{M_{z_k}}(z_k, \mathbf{z}^*_{-k}) \text{ for all } z_k \in Z,\ k \in \{1, ..., n\}. \tag{2}$$

Here, we do not attempt to completely characterise firms' merger decisions, but focus instead on the conditions under which concentrated outcomes can emerge in equilibrium.

Proposition 1 *If $n \in \{2,3\}$, then merger to monopoly can be supported in an SPE for all $\sigma \in (0,1)$. If $n \geq 4$, then there exists a $\widehat{\sigma}(n)$, $\widehat{\sigma}(n) \in (0,1)$, such that merger to monopoly is sustainable if and only if $\sigma \in [\widehat{\sigma}(n), 1)$.*

Proof. See Apendix A. ■

To understand Proposition 1, note first that merger to monopoly from duopoly is always an equilibrium outcome: a monopolist can make at least the same profit per product as a duopolist (by mimicking the duopolists' pricing decisions), and strictly more whenever products are not independent (by raising the price slightly in order to internalise the externalities). This argument breaks down when there are more than two firms (products). Clearly, profit per product is higher under merger to monopoly than under a completely fragmented market structure where each firm offers one product only. However, if a firm deviates unilaterally, it can free ride on its $n-1$ merging rivals. The profit of such a free rider is strictly higher than the profit (per product) prior to the merger game, and may be higher or lower than profit under monopoly. Merger to monopoly will occur if and only if products are sufficiently good substitutes in the sense that price competition will then be sufficiently tough so as to drive down profits whenever firms do not merge to monopoly.[8]

The higher the number of entrants, n, the more "difficult" the merger to monopoly. The reason is that each firm's merger decision becomes less "decisive" as n increases, since – for a given market structure – a free-riding firm is always better off than a

[8]Monopoly profit is also decreasing in the degree of substitutability, σ, since consumers value variety. In the limit as $\sigma \to 1$, industry profit under monopoly converges to the (standard) monopoly profit of a homogeneous goods industry; under all other market structures, industry profit goes to zero.

merging firm, firms have less incentives to merge for higher n.[9] Indeed, it can be shown that $\widehat{\sigma}'(n) > 0$ for $n \geq 4$. Moreover, we get the following result.

Corollary 1 *For any $\sigma \in (0,1)$, there exists a finite threshold value $\widehat{n}(\sigma)$ such that merger to monopoly is sustainable in an SPE if and only if $n < \widehat{n}(\sigma)$.*

Proof. All we need to show is that $\lim_{n\to\infty} \widehat{\sigma}(n) = 1$. Since $\widehat{\sigma}'(n) > 0$ and $\widehat{\sigma}(n) \in (0,1)$ for all $n \geq 4$, it follows that $\lim_{n\to\infty} \widehat{\sigma}(n) \leq 1$. Suppose the assertion is false. Then there exists a $k \in (0,1)$ such that $\lim_{n\to\infty} \widehat{\sigma}(n) < k$. Using (17), this leads to a contradiction. ■

The corollary indicates that it is impossible to sustain merger to monopoly in markets with a sufficiently large number of firms. This holds for any degree of substitutability between products. The following proposition tightens up our predictions.

Proposition 2 *For any $(\sigma, \gamma) \in (0,1)^2$, there exists a finite $n(\sigma;\gamma)$ such that, for all $n \geq n(\sigma;\gamma)$, the market share of each coalition is bounded from above by γ in any equilibrium of the merger game.*

Proof. See Apendix A. ■

Proposition 2 may suggest that mergers will not occur in the limit as the number of firms tends to infinity. This is not the case, however, as illustrated in the following proposition.

Proposition 3 *In equilibrium, there can be at most one single-product coalition. Hence, if $n \geq 2$, then mergers will occur in any equilibrium.*

Proof. This is essentially a corollary of Theorem 1 in Deneckere and Davidson (1985). All we need to show is that if, after relabelling, $z_k = k$ for $k \in \{1,2\}$, and $z_k \geq 3$ for all $k \in \{3,...,n\}$, then firm 1, say, can profitably deviate by joining coalition M_2. This deviation clearly increases the prices of all products in the industry. Decompose the price effect of the proposed deviation into two steps. First, let the outsiders (all coalitions M_k, $k \geq 3$) raise their prices to their new equilibrium values. This will benefit the members of M_2 since it raises the demand for M_2's products. Second, let coalition M_2 raise the prices of its two products to their new equilibrium values. By definition, these price increases must be profitable for the members of M_2. We have thus shown that the proposed deviation by firm 1 is profitable. ■

[9] With a continuum of firms, no firm would want to merge since its own decision does not affect market price, and a firm is clearly worse off by reducing output.

It is worth pointing out that Proposition 3 obtains under the hypothesis of no post-merger entry. A very similar proof can be used to show that, in the present setting, *any* merger is profitable in that it increases the profit of all firms involved in the merger. Nevertheless, as already indicated, concentrated outcomes will not emerge in equilibrium if the number of firms is large.

Entry Stage. We now turn to the determination of n, the number of pre-merger entrants, as a function of market size, S, and entry costs, ϵ. Since we are unable to characterise equilibrium in all ensuing subgames, the equilibrium number of entering firms, $n^*(S/\epsilon)$, cannot be determined. However, it is both possible and useful to compute a lower bound on this number, and to study the limit behaviour of this bound as market size relative to setup costs tends to infinity. For this purpose, it should be kept in mind that, for a given number n of entering firms, the equilibrium profit of any entrant is bounded from below by the profit in the absence of mergers, $S\underline{\pi}(n)$. The reason is that, for given participation decisions of the other $n-1$ firms, an entrant can always choose to form a "coalition" on its own, i.e. not to merge with other firms; this yields a lower bound on its profit. Moreover, the profit of a single-product coalition is minimal if none of the other $n-1$ firms decides to merge.[10] To obtain the lower bound on the number of entrants, we note that $S\underline{\pi}(n)$ is strictly decreasing in n, and strictly increasing in S. Hence, the number of entering firms in the absence of mergers, $\underline{n}(S/\epsilon)$, is the maximum integer n such that $S\underline{\pi}(n) \geq \epsilon$. Moreover, $\underline{n}(S/\epsilon)$ is strictly increasing in S/ϵ, and $\lim_{S/\epsilon\to\infty} \underline{n}(S/\epsilon) = \infty$. Since $n^*(S/\epsilon) \geq \underline{n}(S/\epsilon)$, it follows that $\lim_{S/\epsilon\to\infty} n^*(S/\epsilon) = \infty$. This, in conjunction with Propositions 2 and 3, establishes the following proposition.

Proposition 4 (1.) *For any $(\sigma, \gamma) \in (0,1)^2$, there exists a finite $\overline{(S/\epsilon)}(\sigma;\gamma)$ such that, for all $S/\epsilon \geq \overline{(S/\epsilon)}(\sigma;\gamma)$, the market share of each coalition is bounded from above by γ in any equilibrium of the game.* (2.) *For any S/ϵ sufficiently large, mergers occur in any equilibrium.*

[10] From (16), the lower bound on profits is therefore given by

$$S\underline{\pi}(n) = S \cdot \frac{(1-\sigma)\left[1-\sigma+(n-1)\sigma\right]}{2\left[1-\sigma+n\sigma\right]\left[2(1-\sigma)+(n-1)\sigma\right]^2}.$$

Similarly, an upper bound on the equilibrium number of entrants can easily be found by observing that the equilibrium profit of the worst-off entrant is bounded from above by the profit per product under merger to monopoly, $S\overline{\pi}(n) = S/\left(8\left[1-\sigma+n\sigma\right]\right)$, holding fixed the number n of entrants. Hence, the equilibrium number of entering firms is bounded from above by $\left[S - 8\varepsilon(1-\sigma)\right]/(8\varepsilon\sigma)$.

Proposition 4 states the central prediction for exogenous sunk cost industries. It is impossible to sustain very concentrated outcomes (in the sense of high concentration ratios) in large exogenous sunk cost industries. More precisely, the *upper bound to concentration* converges to zero as market size goes to infinity.

2.2 Monopolisation in Endogenous Sunk Cost Industries

We now turn to the analysis of the limits of monopolisation in endogenous sunk cost industries, where the level of sunk costs is endogenously determined by firms' investment decisions.

2.2.1 The Model

The endogenous sunk cost model differs from the above exogenous sunk cost model in that firms can invest in R&D or advertising so as to increase the consumers' willingness-to-pay for their products. As before, there are n_0 potential entrants, each equipped with the know-how to produce one distinct substitute good, and S identical consumers. Using our previous notation, the utility function is now given by

$$U(\mathbf{x}; H; \mathbf{u}) = \sum_{k=1}^{\infty} \left(x_k - \frac{x_k^2}{u_k^2} \right) - 2\sigma \sum_{k=1}^{\infty} \sum_{l<k} \frac{x_k}{u_k} \frac{x_l}{u_l} + H, \tag{3}$$

where u_k, $u_k \in [1, \infty)$ is the perceived quality of substitute good k. If $u_k = 1$ for all k, then (3) reduces to the utility function of the exogenous sunk cost model (1). It is easy to verify that an increase in u_k strictly increases utility whenever $x_k > 0$; that is, consumers value quality.

The timing of this four-stage game is as follows. At the first stage, the n_0 potential entrants decide whether to enter the market. The entry cost is denoted by ϵ. Again, we assume n_0 to be sufficiently large. At the second stage, the firms that have decided to enter at the preceding stage play the same simultaneous-move coalition formation game as in the exogenous sunk cost case. At the third stage, the newly formed coalitions simultaneously choose the qualities of their products by investing in fixed R&D or advertising outlays. The cost of achieving quality u_k, $u_k \in [1, \infty)$, for good k is given by

$$F(u_k) = F_o \left(u_k^{\beta} - 1 \right), \tag{4}$$

where the parameter β is the elasticity of the investment cost function; we assume $\beta > 2$. Hence, if a firm decides to offer a basic-quality of the good only ($u_k = 1$), it does not have to pay further investment costs. At the final stage, the coalitions simultaneously compete in prices; production costs are assumed to be zero. Note that this model would coincide with the exogenous sunk cost model if we imposed the additional constraint $u_k = 1$ for all k.

2.2.2 Equilibrium Analysis

Solving for the Nash equilibrium in prices is more tedious in the endogenous sunk cost model than in the exogenous sunk cost model. The reason is that products of sufficiently low quality have zero sales in equilibrium; the equilibrium price of these products is not uniquely defined (except possibly for those goods with *just* zero sales). Moreover, a good of a certain quality might not be produced in equilibrium, while a good of a lower quality, owned by a different coalition, has positive sales. This is due to "portfolio effects".

More importantly, it is very difficult to solve for equilibria at the investment stage, due to the following problems: *i)* A multiproduct coalition may find it optimal to invest in a subset of its products only. The equilibrium number of products it offers depends on the substitutability of products (σ), the effectiveness of R&D and advertising (β), and on the details of the coalition structure. For instance, a monopolist may offer less products than a coalition that has less members but faces rivals. *ii)* Even for a fixed number of products, it is not possible to solve the first-order conditions for quality explicitly, unless the coalition structure is symmetric. Moreover, the first-order conditions are not sufficient conditions for a global profit maximum; boundary solutions are endemic. *iii)* Multiple equilibria at the investment stage may arise even under symmetric coalition structures.

In contrast to the exogenous sunk cost case, the size of a coalition and the sizes of rival coalitions are no longer sufficient to summarise a coalition's equilibrium profit in the ensuing subgame: the profits of coalitions of equal size may differ since equilibria at the investment stage are often asymmetric.

Therefore, instead of solving the game, we aim to find a lower bound on the one-firm concentration ratio that can arise in equilibrium. Clearly, no equilibrium market structure can be more fragmented (in terms of the market share of the largest coalition) than the market structure that would arise in the absence of mergers.[11]

[11] Denote by $\widehat{n}$ the equilibrium number of entrants in the constrained game (where mergers are not allowed), and suppose that, in the equilibrium of the unconstrained game (where mergers are allowed), the largest coalition has $\widehat{m}$, $\widehat{m} \geq 2$, active members. For simplicity, let the relative number of products be

Most-Fragmented Market Structure: The Constrained Game. To find a lower bound on concentration in our endogenous sunk cost game, we analyse a "constrained game" instead, which is identical to the four-stage game described above, except that firms are not allowed to merge. That is, in the constrained game, each entrant k is constrained to set $z_k = k$ at the second stage. The equilibrium analysis of the constrained is rather involved and is deferred to Appendix B.

In Appendix B, we show that the equilibrium number of entrants is bounded from above, no matter how large the market. That is, even in the absence of mergers, concentration in endogenous sunk cost industries cannot become arbitrarily small by increasing the size of the market. This implies that Proposition 4 does not hold in endogenous sunk cost industries. This is the "nonfragmentation result" (Shaked and Sutton, 1987), according to which fragmented outcomes cannot be supported as equilibrium outcomes in endogenous sunk cost industries. It is one of the fundamental findings in the theory of industrial market structure. What is the intuition behind the result? Suppose nonfragmentation would not obtain and the number of firms tends to infinity in the limit as the market becomes large. Hence, in large markets, each firm's market share is small. Since a firm's revenue must cover at least its fixed investment outlays, this implies that each firm cannot spend much on R&D or advertising. Now, this cannot be an equilibrium as there would be a profitable deviation for some firm, which consists in raising quality by outspending its rivals and capturing a large share of the market. Therefore, in the limit as market size becomes large, firms' equilibrium investment outlays rise proportionally with market size, and the number of firms the market can support becomes independent of market size.

The key question, not addressed by Shaked and Sutton, is whether *arbitrarily* concentrated outcomes can be sustained. In Appendix B, we show that the limit number of entering firms is decreasing in the substitutability parameter σ. This is for two reasons. First, the higher is σ, the less variety is offered by the market, and hence the less consumers spend on the goods offered in this industry, holding prices fixed. Second, the

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the measure of a coalition's market share. Then, for the one-firm concentration ratio in the unconstrained game to be lower than $1/\widehat{n}$, the equilibrium number of entrants would have to be larger than $\widehat{m}\widehat{n}$. It is possible to show that entry of more than $\widehat{m}\widehat{n}$ cannot be profitable. To see this, note that the profit of the worst-off entrant will be maximal (under the constraint that concentration is not more than $1/\widehat{n}$) if the $\widehat{m}\widehat{n}$ single-product entrants each formed $\widehat{n}$ coalitions of size $\widehat{m}$. In the constrained game, however, the post-entry profit per product of any of these coalitions is lower than that of any of the $\widehat{n}$ single-product entrants.

larger the degree of substitutability between goods, the tougher is price competition, and thus the lower are profit margins. We show that that there is a nondegenerate interval of values of σ (close to 1) for which the equilibrium number of entering firms is one, no matter how large the market. The number of entrants is increasing in β since the more convex the investment cost function, the less firms will outspend each other in equilibrium. We consider two limit cases. First, if $\beta \to \infty$, then for no value of σ is the limit number of entrants finite. This limit can be interpreted as the "exogenous sunk cost case", where every active firm chooses quality level 1 in equilibrium, and the number of firms grows without bound as market size increases. Second, if $\beta \to 2$, then for all values of σ is the limit number of entering firms equal to one. This limit may be dubbed the "natural monopoly case". Our results may be summarised in the following proposition.

Proposition 5 *In the constrained game, where firms are not allowed to merge, the equilibrium number of firms remains finite, no matter how large the market. In particular, if products are sufficiently good substitutes (σ close to 1) or investment is sufficiently effective (β close to 2), only one firm will enter the market, even as market size (relative to setup costs) tends to infinity.*

In the unconstrained game, where firms are allowed to merge, more concentrated outcomes will emerge in equilibrium, and more firms will enter the market, than in the constrained game. We have shown that the most fragmented market structure in endogenous sunk cost industries may involve arbitrarily concentrated outcomes in large markets. The empirical prediction for endogenous sunk cost industries may thus be summarised as follows.

Corollary 2 *In endogenous sunk cost industries, arbitrarily high one-firm sales concentration ratios may be supported in equilibrium, even in the limit as market size tends to infinity. That is, the upper bound to concentration does not decrease with market size.*

It should be pointed out that this prediction is *not* a mere consequence of the nonfragmentation result, according to which arbitrarily fragmented market structures cannot be sustained in large endogenous sunk cost industries. Our corollary states that it is possible to support *arbitrarily* concentrated outcomes; this holds independently of the size of the market and the level of setup costs.

3 Ex-Post Entry and the Limits to Concentration

To what extent does the possibility of "ex-post entry" constrain industry structure and, in particular, the emergence of concentrated outcomes? As mentioned in the Introduction, post-merger entry prevents firms from successfully monopolising markets through horizontal mergers; see Stigler (1950). In fact, we intend to show that the possibility of ex-post entry prevents the emergence of concentrated outcomes in large exogenous sunk cost industries. Moreover, by allowing for ex-post entry, our analysis can be made very general along several dimensions, and previous assumptions on coalition formation and product selection can be relaxed.

Indeed, the main predictions of Section 2 regarding the relationship between market size and the upper bound to concentration were derived under quite special assumptions. First, we modelled coalition formation, i.e. mergers, as an open membership game. Second, we assumed that multiproduct firms can only emerge through mergers; firms are not allowed to choose the number of products they would like to offer. Due to the first assumption, it is an open question whether the "instability" of concentrated outcomes is a consequence of coordination failures, which may or may not occur under different assumptions on coalition formation. After all, for a given number of firms in the industry, joint profits are maximised under monopoly. The second assumption implies that the only way to sustain a concentrated outcome is through mergers. This is clearly both unrealistic and restrictive, especially in the presence of antitrust laws.

To show that our predictions do not depend on such assumptions, we apply an equilibrium concept, due to Sutton (1997), which is defined in the space of observable outcomes. This equilibrium concept involves two rather weak assumptions, "viability" (no firm makes losses) and "stability" (there is one smart agent who would fill a profitable opportunity in the market), both of which are implied by subgame perfection. The key feature of the assumed extensive form of the game is that there is some penultimate stage (before competition takes place) where new firms may enter the market, taking as given the actions of the incumbents. This formalises Stigler's notion of "post-merger entry". Crucially, it allows us to be quite general regarding firms' action space and the details of the extensive form of the game.

3.1 The Model

There are n_0 firms that can take actions at certain specified stages. Each firm's action space (in the first T stages) is denoted by A (which may differ across firms). Actions may include decisions regarding entry, the number of products, mergers, product quality (in the case of endogenous sunk cost industries), takeover bids, and so on. Firm i's actions (in the first T stages of the game) are summarised by the vector $\mathbf{a}_i$, $\mathbf{a}_i \subset A$. Each firm may decide not to enter the market, i.e. to choose the "null action", denoted by $\mathbf{a}_i = \emptyset$. The outcome of the game can then be described by the n_0-tuple $(\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_{n_0})$. Suppose n, $n \in \{1, ..., n_0\}$ firms decide to enter the market, i.e. to choose a non-null action. Then, deleting all inactive firms and relabelling the remaining active firms, yields the n-tuple

$$\mathbf{a} \equiv (\mathbf{a}_1, \mathbf{a}_2, ..., \mathbf{a}_n),$$

which is referred to as a *configuration*.

The total payoff (profit) of firm i from the set of actions $\mathbf{a}_i$, when rivals' actions are given by $\mathbf{a}_{-i} \equiv (\mathbf{a}_1, ..., \mathbf{a}_{i-1}, \mathbf{a}_{i+1}, ..., \mathbf{a}_n)$, is written as

$$\overline{\Pi}(\mathbf{a}_i; \mathbf{a}_{-i}).$$

If firm i decides not to enter the market ($\mathbf{a}_i = \emptyset$), then its payoff is zero:

$$\overline{\Pi}(\emptyset; \mathbf{a}_{-i}) = 0.$$

The function $\overline{\Pi}(\mathbf{a}_i; \mathbf{a}_{-i})$ summarises not only the final-stage profits but also possible costs from taking the set of actions $\mathbf{a}_i$ (e.g. costs of introducing a new product) as well as payments between firms (resulting from merger or takeover decisions).

To exclude nonviable markets, we assume that there is some action $\mathbf{a}_0$, $\mathbf{a}_0 \neq \emptyset$, such that

$$\overline{\Pi}(\mathbf{a}_0; \emptyset) > 0. \tag{5}$$

Furthermore, the number of potential entrants, n_0, is assumed to be sufficiently large such that if all firms choose to enter the market, then at least one firm incurs a loss. The idea is that entering the market requires a minimum setup cost of ϵ, and the sum of final-stage payoffs is bounded from above by $(n_0 - 1)\epsilon$.

All we have to specify about the extensive form of the game is the following. There are T stages at which firms can enter the market and take actions; associated with these actions are certain costs and payments between firms. Moreover, at stage $T + 1$, competition between firms takes place. All payoffs are summarised by the reduced-form payoff function $\overline{\Pi}(\cdot;\cdot)$. Firm i is free to take actions at any date t, $t \in \{t_i, t_i + 1, ..., T\}$, where $t_i \in \{1, ..., T\}$ is firm i's "date of arrival". This allows for "first-mover advantages". The important feature of the extensive form is that there is some penultimate stage, T, at which firms take actions simultaneously, and new firms can enter the market, before firms engage in price competition. We do not allow for actions that effectively condition on the outcome of this penultimate stage.

A configuration $\mathbf{a}^*$ is called an *equilibrium configuration* if the following two conditions are satisfied:

(i) (viability) for all firms i,

$$\overline{\Pi}(\mathbf{a}_i^*; \mathbf{a}_{-i}^*) \geq 0,$$

(ii) (stability) there is no set of actions $\mathbf{a}_{n+1}$ such that entry is profitable. That is,

$$\overline{\Pi}(\mathbf{a}_{n+1}; \mathbf{a}^*) \leq 0.$$

Condition (i) requires that none of the firms make a loss in equilibrium, while condition (ii) says that if the market offers a profitable opportunity, then there is some smart agent who will fill it. Both conditions are consistent with boundedly rational agents who do not fully maximise their payoffs. Moreover, both conditions are implied by subgame perfection. To see this, note that if the viability condition were not satisfied in a candidate SPE, then a firm could profitably deviate by choosing the null action ("do not enter"), and make zero profit. Similarly, if the stability condition were not satisfied, then an inactive firm could profitably deviate by entering the market at stage T. We thus have the following "inclusion" property.

Proposition 6 (Sutton 1997) *Any outcome that can be supported in an SPE in pure strategies is an equilibrium configuration.*

Needless to say, the concept of an equilibrium configuration will only have bite in empirical applications if the conditions of viability and stability can be expressed in the space of observable outcomes. This is indeed the case if firms' actions merely consist in choosing the number of products (exogenous sunk cost industries) or product quality (endogenous sunk cost industries). If we want to allow for side payments between firms, however, then these conditions have to be re-formulated in the space of observables.

So far, we have been silent about the nature of competition at stage $T+1$ and about the costs of taking actions in stages 1 to T. In the following, we discuss these issues in turn for exogenous and endogenous sunk cost industries.

Exogenous Sunk Cost Industries. In this case, a profile of firms' actions, i.e. a configuration, $\mathbf{a}$, induces a profile of products (a coalition structure)

$$\mathbf{m} \equiv (m_1, m_2, ..., m_l),$$

$l \in \{1, ..., n\}$, where m_i is the number of products in firm (or coalition) i's portfolio. The demand structure is as in Section 2.1, and firms (coalitions) are assumed to compete in prices at the ultimate stage. Suppose the total number of products offered in the industry is given by $\overline{m} \equiv \sum_{j=1}^{l} m_i$. Then, provided it has chosen to enter the market, firm i's profit from the final price competition stage is given by $S\Pi(m_i; \mathbf{m}_{-i})$, which can be derived from equation (16). The setup cost per product is denoted by ϵ.

Suppose now that $\mathbf{a}^*$ forms an equilibrium configuration, which induces the profile of products, $\mathbf{m}^*$. The viability and stability conditions for exogenous sunk cost industries may be expressed in the space of observables (i.e. in the space of profiles of product numbers) as follows:

(i') For all $i \in \{1, ..., l\}$,

$$S\Pi\left(m_i^*; \mathbf{m}_{-i}^*\right) - m_i^* \epsilon \geq 0.$$

(ii') There does not exist an m_{n+1}, $m_{n+1} \in \{1, 2, ...\}$, such that

$$S\Pi\left(m_{n+1}; \mathbf{m}^*\right) - m_{n+1}\epsilon > 0.$$

Condition (ii') is slightly weaker than (ii) in that we restrict the actions of an additional entrant to choosing the number of its products. To understand condition (i'), note that $S\Pi\left(m_i^*; \mathbf{m}_{-i}^*\right) - m_i^* \epsilon$ is an upper bound on firm i's total payoff under the assumption that the implicit price (in the event of coalition formation: the profit share), or the explicit price (in the event of takeovers: the takeover bid), of acquiring a product from another firm is at least the setup cost per product, ϵ. (The rationale for this assumption is to exclude cross-subsidies between firms, e.g. one firm sells a product to a rival below cost. This assumption is somewhat stronger than the viability condition, which only requires a firm not to make a loss on its combined activities.) Conditions (i') and (ii') coincide with (i) and (ii) if firms' actions merely consist in selecting the number of their products. In the equilibrium analysis below we show that these two rather weak requirements are nevertheless powerful enough to obtain strong empirical predictions.

Endogenous Sunk Cost Industries. Here, application of the equilibrium concept proceeds similarly to the case of exogenous sunk cost industries. A configuration $\mathbf{a}$ induces a profile of qualities

$$\mathbf{u} \equiv (\mathbf{u}_1, \mathbf{u}_2, ..., \mathbf{u}_l),$$

$l \in \{1, ..., n\}$, where $\mathbf{u}_i$ is the vector of qualities offered by firm (coalition) i. Firm i's final stage profit is denoted by $S\Pi(\mathbf{u}_i; \mathbf{u}_{-i})$. We use our earlier demand system and assume that firms compete in prices, so that $S\Pi(\mathbf{u}_i; \mathbf{u}_{-i})$ is the Nash equilibrium profit associated with the final stage in the (unconstrained) game of Section 2.2. The cost of investment is given by equation (4); again, the entry cost per product is ϵ.

Suppose now that $\mathbf{a}^*$ forms an equilibrium configuration, which induces the profile of qualities $\mathbf{u}^*$. Denote by M_i the set of products in firm i's portfolio at the end of the game. The viability and stability conditions can then be expressed in the space of observables as follows:

(i") For all i, $i \in \{1, ..., l\}$,

$$S\Pi\left(\mathbf{u}_i^*; \mathbf{u}_{-i}^*\right) - \sum_{k \in M_i} \left(F_o u_k^\beta + \epsilon\right) \geq 0.$$

(ii") There does not exist a vector of qualities, $\mathbf{u}_{n+1}$, $\mathbf{u}_{n+1} \neq \mathbf{0}$, such that

$$S\Pi\left(\mathbf{u}_{n+1}; \mathbf{u}^*\right) - \sum_{k \in M_{n+1}} \left(F_o u_k^\beta + \epsilon\right) > 0.$$

Again, if firms' actions merely consist in choosing product qualities (and the number of products), then these two conditions coincide with (i) and (ii).

3.2 Equilibrium Configurations

We now investigate whether concentrated outcomes can be sustained as equilibrium configurations in large exogenous and endogenous sunk cost industries, respectively.

3.2.1 Exogenous Sunk Cost Industries

Before turning to the exogenous sunk cost case, some further notation is called for. We denote firm i's final-stage profit *per product* by $S\pi(m_i; \mathbf{m}_{-i})$, i.e. $\pi(m_i; \mathbf{m}_{-i}) \equiv \Pi(m_i; \mathbf{m}_{-i})/m_i$.

For a given number of products, industry profits are clearly maximised under monopoly. Does this imply that monopoly will endogenously emerge in equilibrium? Not necessarily. When addressing endogenous horizontal mergers, we found that by staying out of a coalition, a firm may be better off than by joining. In fact, we showed (slightly abusing notation) that

$$\pi(1; \overline{m} - 1) > \pi(\overline{m}; 0) \text{ for } \overline{m} \text{ sufficiently large.} \tag{6}$$

Therefore, in an open membership game, firms will not endogenously merge to monopoly if the number of firms in the industry is sufficiently large.

This "inefficient" outcome may be viewed in terms of some coordination failure, i.e. if firms were allowed to renegotiate coalition formation and make side payments, monopoly could be achieved. However, any renegotiation should be modelled explicitly, and it is *a priori* not clear whether such renegotiation would lead to an efficient outcome. Anticipating renegotiation, firms would have even fewer incentives to merge prior to renegotiation. More importantly, renegotiation has no impact unless it takes place after all entry has occurred. Our model formalises this idea: there is a penultimate stage at which firms may renegotiate earlier agreements and, simultaneously, new entry may occur.

We have already shown that (6) holds. It is straightforward to obtain a stronger result[12]:

$$\pi(1; \overline{m}) > \pi(\overline{m}; 0) \text{ for } \overline{m} \text{ sufficiently large.} \tag{7}$$

Ex-post entry, along with (7), are sufficient to imply that monopoly will not occur in large markets. To see this, suppose to the contrary that $(\overline{m}, 0)$ is sustainable as an equilibrium configuration. Viability and stability require

$$\pi(\overline{m}; 0) \geq \epsilon/S$$

[12]Observe that

$$\pi(\overline{m}; 0) = \frac{1}{8\left[1 - \sigma + \overline{m}\sigma\right]}$$

and

$$\pi(1; \overline{m}) = \frac{(1-\sigma)\left[1 + (\overline{m}-1)\sigma\right]\left[2 + \overline{m}\sigma\right]^2}{2[1 + \overline{m}\sigma]\left[4 + 4(\overline{m}-1)\sigma - \overline{m}\sigma^2\right]^2}.$$

Taking the limit as $\overline{m}$ tends to infinity, we obtain

$$\lim_{\overline{m} \to \infty} \{\pi(\overline{m}; 0) - \pi(1; \overline{m})\} < 0,$$

which proves the claim.

and

$$\pi(1;\overline{m}) \leq \epsilon/S, \tag{8}$$

which imply

$$\pi(\overline{m};0) \geq \pi(1;\overline{m}). \tag{9}$$

Let $\widehat{m}(S/\epsilon)$ denote the maximum integer such that $\pi(1;\widehat{m}(S/\epsilon)) \geq \epsilon/S$. Since $\pi(1;\overline{m})$ is strictly decreasing in $\overline{m}$, and $\lim_{\overline{m}\to\infty} \pi(1;\overline{m}) = 0$, we have $\widehat{m}(S/\epsilon) \to \infty$ as $S/\epsilon \to \infty$. Configuration $(\overline{m},0)$ satisfies the stability condition (8) if $\overline{m} \geq \widehat{m}(S/\epsilon)$. From equation (7), it then follows that $\pi(1;\overline{m}) > \pi(\overline{m};0)$ in sufficiently large markets. But this is in contradiction to (9). We have thus shown that it is possible to exclude monopoly outcomes in large exogenous sunk cost industries. In fact, we obtain a much stronger result: the upper bound to concentration goes to zero as market size tends to infinity.

Proposition 7 *For any $(\sigma,\gamma) \in (0,1)^2$, there exists a threshold level $\left(\overline{S/\epsilon}\right)(\sigma;\gamma)$ such that for all $S/\epsilon \geq \left(\overline{S/\epsilon}\right)(\sigma;\gamma)$, the market share of the largest firm is bounded from above by γ in any equilibrium configuration.*

Proof. See Appendix A. ■

The proposition shows that the predictions we have derived regarding exogenous sunk cost industries are robust; they do not depend on the details of the extensive form of the game, provided we allow for ex-post entry.

Proposition 7 characterises equilibrium configurations. Existence of an equilibrium configuration in the exogenous sunk cost case can be shown as follows. Let $\mathbf{1}_n$ denote the n-tuple $(1,1,...,1)$. Then, define $n(\epsilon/S)$ as the maximum integer n such that $\pi(1;\mathbf{1}_{n-1}) \geq \epsilon/S$. From assumption (5), $n(\epsilon/S) \geq 1$. We claim that the maximally fragmented market structure $\mathbf{1}_{n(\epsilon/S)}$ can be sustained as an equilibrium configuration. To see this, note that $\mathbf{1}_{n(\epsilon/S)}$ satisfies the viability condition by definition. Moreover, for any $m \in \{1,2,...\}$, $\pi\left(m;\mathbf{1}_{n(\epsilon/S)}\right) \leq \pi\left(1;\mathbf{1}_{n(\epsilon/S)}\right) < \epsilon/S$, so that the stability condition holds as well. This proves the claim.

3.2.2 Endogenous Sunk Cost Industries

According to our rather specific endogenous horizontal merger model of Section 2.2, in endogenous sunk cost industries, very concentrated outcomes – even monopoly – may be sustained, no matter how large the market. We are now in a position to show that this prediction carries over to the current setting.

Proposition 8 *If products are sufficiently good substitutes (σ close to 1), or investment in quality enhancement is sufficiently effective (β close to 2), monopoly can be sustained in an equilibrium configuration. This holds independently of the level of market size relative to setup costs, provided the market is not too small so as to support at least one firm.*

Proof. Using the notation of Appendix B, suppose the candidate monopolist offers one product, which is of quality $\overline{u}(1)$. We have already shown that if $\sigma \in (\sigma_1(\beta), 1))$, then, even in the limit as market size relative to setup costs tends to infinity, entry is unprofitable for a firm which is restricted to offering only one product. Moreover, the monopolist makes positive profits, provided market size (relative to setup costs) is not too small. (Note that the level of σ has no impact on the profit of a single-product monopolist.) The extension of the result to the case of a multiproduct entrant is proved in two steps: first, by showing that a multiproduct entrant optimally chooses the same quality for all of its products, and second, by observing that the final-stage profit *per product* is decreasing in the number of own products, holding quality fixed. ■

Although the possibility of ex-post entry works against the emergence of concentrated outcomes, there are several reasons why monopoly may be sustained in equilibrium for a larger set of parameter values in the present model than in the model of Section 2.2. First, entry deterrence through quality investment (and product proliferation) is now consistent with the concept of an equilibrium configuration. In contrast, in the previous model, entry deterrence is not consistent with subgame perfection since all firms invest simultaneously in quality. Second, the present equilibrium concept no longer requires the candidate monopolist to maximise profits. In particular, the monopolist may overinvest in quality relative to the profit-maximising level. Third, in the earlier model, a multiproduct firm can only emerge through mergers, and mergers are, potentially, subject to coordination failures. In the present setup, such coordination failures are muted; for instance, the candidate monopolist may simply select the number of products so as to deter entry.

It is straightforward to show the existence of an equilibrium configuration for *any* set of parameter values satisfying (5). Using the notation of Appendix B, suppose n firms offer a product of quality $\overline{u}(n)$ each. Hence, each firm's profit (gross of entry cost ϵ) is given by $\Pi^*(n)$. Let $\overline{n}(\epsilon)$ denote the largest integer n such that $\Pi^*(n) \geq \epsilon$; from (5), $\overline{n}(\epsilon) \geq 1$. Then, we claim that $(\overline{u}(\overline{n}(\epsilon)), ..., \overline{u}(\overline{n}(\epsilon)))$ can be sustained as an equilibrium configuration. Indeed, the viability condition is satisfied by construction. Moreover, as we have already shown, additional entry by a single-product firm cannot be profitable. Following the argument in the proof of Proposition 8, entry by a multiproduct firm must be unprofitable as well. Hence, the stability condition also holds.

4 Related Literature

This paper is closely related to several strands in the I.O. literature. First, it belongs to the game-theoretic literature on industrial market structure and, in particular, on the relationship between market size and concentration. Important works in this literature are Sutton (1991, 1998), dealing with advertising-intensive and R&D-intensive industries, respectively.[13] Much of Sutton's work is concerned with the stability of fragmented outcomes in large markets. The instability of such outcomes in models of pure vertical product differentiation was first shown by Shaked and Sutton (1983), extending earlier work by Gabszewicz and Thisse (1980). Shaked and Sutton (1987) later found that this result generalises to industries where firms can effectively raise consumers' willingness-to-pay by investing in some fixed outlays. In these papers, nonconvergence is analysed in the context of static stage games. The robustness of these results to the existence of collusive underinvestment equilibria in dynamic investment games has been shown by Nocke (1998). In the present paper, we develop this literature further by examining the stability of concentrated outcomes in large markets. Following Sutton (1991), we distinguish between exogenous and endogenous sunk cost industries, building on the insights of this earlier work.

Second, in this paper, firms attempt to monopolise markets by horizontal mergers. The I.O. literature on horizontal mergers can be roughly divided into two strands: exogenous and endogenous mergers. The first strand is mainly concerned with the profitability and welfare consequences of a given horizontal merger by two or more firms; see e.g. Salant, Switzer and Reynolds (1983), Perry and Porter (1985), Deneckere and Davidson (1985), Levin (1990), and Farrell and Shapiro (1990).[14] Most of these papers analyse mergers in a homogeneous goods industry under Cournot competition. Salant, Switzer and Reynolds (1983) demonstrated that mergers tend to be unprofitable in such a setting, provided there are no efficiency gains and the merger does not lead to almost

[13] Important empirical articles are Bresnahan and Reiss (1991) and Berry (1992).

[14] Some papers also study the effects of exogenous horizontal mergers on collusion; see Davidson and Deneckere (1984) and Compte, Jenny and Rey (1996).

complete monopolisation. Deneckere and Davidson (1985) showed that under Bertrand competition (with differentiated products), this result no longer holds. In the exogenous sunk cost case, we use essentially the same multiproduct demand system as Deneckere and Davidson. The advantage of such a setting is that mergers are conceptually well defined; in contrast, in a homogeneous goods model with constant returns-to-scale technology, as in Salant, Switzer and Reynolds (1983) and Kamien and Zang (1990), a merger by m firms is equivalent to a reduction in the number of players by $m-1$.

Our paper is even more closely related to the as yet underdeveloped literature on endogenous horizontal mergers, originating from Stigler's (1950) insight that firms may not want to participate in a merger since they might prefer to free ride on the merging firms' efforts to restrict output. Hence, the profitability of a given merger (relative to no merger at all) is in general not sufficient for a merger to occur in a noncooperative equilibrium. Using a homogeneous goods Cournot model with constant returns-to-scale technology, Kamien and Zang (1990) were the first to formally analyse endogenous horizontal mergers.[15] In such a setting, merger to monopoly, although maximising industry profits, does not emerge in equilibrium if the number of firms, n, is sufficiently large. By not participating in the merger of its $n-1$ rivals, a firm can ensure itself the duopoly profit. If the monopoly profit is k times the duopoly profit, then merger to monopoly obtains only if $n \leq [k]$, where $[k]$ is the integer part of k. From Salant, Switzer and Reynolds' (1983) analysis, it is well known that a merger which falls (significantly) short of monopoly is not profitable. Not surprisingly, then, for n sufficiently large, mergers do not occur at all in equilibrium.[16] The main differences between Kamien and Zang (1990) and our paper are the following. First, we analyse a multiproduct demand system in which mergers are not merely a reduction in the number of firms. Second, we consider the case of price competition where *any* merger is profitable in that it increases the joint profit of the participating firms; see Deneckere and Davidson (1985). Indeed, we show that mergers will occur in equilibrium, even in the limit as n tends to infinity. The open question we address is whether concentrated outcomes can endogenously occur in equilibrium. Third, and most importantly, we introduce several features which allow us

[15]Earlier, Deneckere and Davidson (1985) discussed firms' incentives to merge, but did not formally analyse coalition formation. Later work includes Kamien and Zang (1991, 1993) and Gowrisankaran (1999) .

[16]Kamien and Zang (1990) show that if merged entities are allowed to partially "demerge" after the merger (by forming independent subunits), then mergers tend to be more profitable. But again, merger to monopoly will not obtain if n is sufficiently large.

to make empirically testable predictions: the degree of horizontal product differentiation (σ), the distinction between exogenous and endogenous sunk cost industries, and market size relative to setup costs (by assuming free but costly entry). Finally, we show that our predictions do not depend on the details of the coalition formation game if the notion of ex-post entry is introduced. In fact, the effects of post-merger entry have been highly neglected in the literature.

Some of the insights of the endogenous horizontal merger literature have been anticipated by studies on cartel stability and explicit collusion; see, for instance, Selten (1973), d'Aspremont et al. (1983), and Nocke (1999a). In fact, these models are formally equivalent to endogenous horizontal merger models. Conversely, our results may also be significant for the literature on cartel stability. Finally, our paper is related to the literature on endogenous (noncooperative) coalition formation (e.g. Hart and Kurz, 1983; Bloch, 1996; Yi, 1997) and multiproduct oligopoly (e.g. Champsaur and Rochet, 1989, and Shaked and Sutton, 1990).

5 Conclusion

This paper constitutes an attempt to sharpen the predictions of the game-theoretic literature on industrial market structure. Sutton (1991), who considered the relationship between market size and concentration, showed that *fragmented outcomes* can in general be sustained in large exogenous sunk cost industries, but not in endogenous sunk cost industries. In our paper, we ask whether it is possible to predict the kinds of industries in which *concentrated outcomes* can and cannot be sustained.

In the first part of the paper, we have analysed an endogenous horizontal merger model with free but costly entry. We have shown that it is impossible to sustain concentrated outcomes in large exogenous sunk cost industries. More precisely, the upper bound to the one-firm concentration ratio goes to zero as market size (relative to setup costs) tends to infinity. In contrast, in endogenous sunk cost industries, where firms can invest in some fixed R&D or advertising outlays to increase the (perceived) quality of their products, *arbitrarily* concentrated outcomes can be sustained even in the absence of mergers, no matter how large the market.

In the second part of the paper, we have formalised Stigler's notion of post-merger entry. We have shown that the details of the extensive form of the game do not matter, once we allow for ex-post entry. In particular, we do not have to commit to any particular merger or coalition formation game and can allow for side payments between firms and endogenous product choice. Most importantly, the main predictions of the first part of the paper carry over to this flexible framework. That is, the predictions on the upper bound to concentration obtain independently of the details of the extensive form, provided that ex-post entry can occur.

How robust are the predictions of this paper? It can be shown that our conclusions do not hinge on the assumption of price competition. In fact, under quantity competition, the incentive to free ride on rivals' efforts to restrict output is larger than under price competition, so that it is more difficult to obtain concentrated outcomes in exogenous sunk cost industries. The potential emergence of highly concentrated outcomes in large endogenous sunk cost industries is not affected. In any event, more research is called for regarding the robustness of our results.

Appendix A: Proofs

Proof of Lemma 1. Since $U(\mathbf{x}; Y - \sum_k p_k x_k)$ is strictly concave in $\mathbf{x}$, there exists a unique utility-maximising consumption bundle, given any price vector $\mathbf{p}$. That is, each consumer has a well-defined demand function for good k, $d_k(\mathbf{p})$; market demand is $D_k(\mathbf{p}) = S d_k(\mathbf{p})$. Recall that, by assumption, $Y > \sum_k p_k d_k(\mathbf{p})$ in equilibrium. Hence, if $d_k(\mathbf{p}) > 0$, demand for good k is implicitly defined by the first-order condition

$$1 - 2(1-\sigma)d_k(\mathbf{p}) - 2\sigma \sum_{l=1}^{n} d_l(\mathbf{p}) - p_k = 0.$$

Relabel products in increasing order of price, i.e. $p_k \leq p_{k+1}$ for all $k \in \{1, ..., n-1\}$. Define the integer $\overline{n}(\mathbf{p})$ as follows. If $p_1 > 1$, then $\overline{n}(\mathbf{p}) = 0$; otherwise, let $\overline{n}(\mathbf{p})$ be the largest integer v, $v \leq n$, such that $(1-\sigma)(1-p_v) - v\sigma\left(p_v - (1/v)\sum_{l=1}^{v} p_l\right) \geq 0$. Demand (per consumer) for good k can now be written as

$$d_k(\mathbf{p}) = \frac{(1-\sigma)(1-p_k) - \overline{n}(\mathbf{p})\sigma\left(p_k - \frac{1}{\overline{n}(\mathbf{p})}\sum_{l=1}^{\overline{n}(\mathbf{p})} p_l\right)}{2(1-\sigma)\left[1 - \sigma + \overline{n}(\mathbf{p})\sigma\right]} \tag{10}$$

if $k \leq \overline{n}(\mathbf{p})$, and $d_k(\mathbf{p}) = 0$ otherwise. Note that, although $\overline{n}(\mathbf{p})$ takes only integer values, $d_k(\mathbf{p})$ is continuous in $\mathbf{p}$.

Let $\mathbf{p}_{M_i}$ denote the vector of coalition M_i's prices, and $\mathbf{p}_{-M_i}$ the price vector of its rivals. Coalition M_i sets the prices of its products so as to maximise its profit:

$$\max_{\{p_k\}_{k\in M_i}} \sum_{k\in M_i} p_k D_k(\mathbf{p}_{M_i}, \mathbf{p}_{-M_i}).$$

Since the demand function has (a finite number of) kinks, a firm's best-reply function is not continuous everywhere. Denote by $\mathbf{p}_{-k} \equiv (p_1, ..., p_{k-1}, p_{k+1}, ..., p_n)$ the vector of prices of all goods other than k. Let us now make three observations. First, there exists a $\overline{p}(\mathbf{p}_{-k})$ such that $d_k(p_k, \mathbf{p}_{-k}) > 0$ if and only if $p_k < \overline{p}(\mathbf{p}_{-k})$. It is easy to see that $\overline{p}(\mathbf{p}_{-k}) > 0$ for all nonnegative $\mathbf{p}_{-k}$, which implies that each firm makes a profit in equilibrium. Second, each product makes positive sales in equilibrium. To see this, suppose that good k, $k \in M_i$, makes no sales. But then, M_i could raise its profit by setting p_k slightly below $\overline{p}(\mathbf{p}_{-k})$, holding all other prices fixed. Third, coalition M_i's profit is continuous in $\mathbf{p}$; it is strictly concave in $\mathbf{p}_{M_i}$ for any $\mathbf{p}_{-M_i}$, provided that prices are such that $p_k < \overline{p}(\mathbf{p}_{-k})$ for all $k \in M_i$. These observations combinded imply that the set of first-order conditions is necessary and sufficient for $\mathbf{p}^*$ to form a Nash equilibrium.[17] Hence, equilibrium price p_k^*, $k \in M_i$, is implicitly defined by

$$1 + \frac{2\sigma}{1-\sigma} \sum_{l \in M_i} p_l^* + \frac{\sigma}{1-\sigma} \sum_{j \notin M_i} p_j^* = 2\left(\frac{1-\sigma+n\sigma}{1-\sigma}\right) p_k^*. \tag{11}$$

Since the left-hand side of (11) is independent of k, it follows that $p_k^* = p_{M_i}^* < \overline{p}(\mathbf{p}_{-k}^*)$ for all $k \in M_i$. That is, a merged entity sets the same price for each of its products.

We can now rewrite the first-order condition as follows

$$p_{M_i}^* = \frac{1-\sigma+\sigma\sum_{j\in Z} m_j p_{M_j}^*}{2(1-\sigma)+(2n-m_i)\sigma}. \tag{12}$$

Multiplying both sides by m_i, and summing over all coalitions, gives

$$\sum_{j\in Z} m_j p_{M_j}^* = \frac{(1-\sigma)\sum_{j\in Z}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}}{1-\sigma\sum_{j\in Z}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}}. \tag{13}$$

Inserting (13) into (12) yields the (unique) equilibrium price of coalition M_i's products:

$$p_{M_i}^* = \frac{1-\sigma}{[2(1-\sigma)+(2n-m_i)\sigma]\left[1-\sigma\sum_{j\in I}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}\right]}. \tag{14}$$

Using (10) and (14), we can now calculate the market demand per product of coalition M_i as

$$D_{M_i}(\mathbf{p}^*) = S\frac{1-\sigma+(n-m_i)\sigma}{2(1-\sigma)[1-\sigma+n\sigma]} \cdot p_{M_i}^*. \tag{15}$$

It is straightforward to verify that $\sum_k p_k^* d_k(\mathbf{p}^*) < 1/8\sigma$, so that the assumption on income indeed ensures that income is higher than the consumer's equilibrium expenditure

[17] In the following, for notational simplicity, we suppress the dependence of strategies on $\mathbf{z}$.

on the n substitute goods.[18] Finally, coalition M_i's equilibrium profit per product can be computed as

$$S\pi^*_{M_i}(\mathbf{z}) = S\frac{(1-\sigma)\left[1-\sigma+(n-m_i)\sigma\right]}{2\left[1-\sigma+n\sigma\right]\left[2(1-\sigma)+(2n-m_i)\sigma\right]^2\left[1-\sigma\sum_{j\in I}\frac{m_j}{2(1-\sigma)+(2n-m_j)\sigma}\right]^2}. \tag{16}$$

■

Proof of Proposition 1. Let $S\pi(m; n-m)$ denote the profit per product of a coalition with m members, facing a single nonempty rival coalition with $n-m$ members. Merger to monopoly can be sustained in equilibrium if and only if

$$\pi(n;0) \geq \pi(1; n-1).$$

Using (16), this condition can be rewritten as

$$\begin{aligned} &\left[4(1-\sigma)^2 + 4n\sigma(1-\sigma) + 3(n-1)\sigma^2\right]^2 \\ &-4(1-\sigma)\left[1-\sigma+(n-1)\sigma\right]\left[2(1-\sigma)+(n+1)\sigma\right]^2 \;\geq\; 0, \end{aligned}$$

which simplifies to

$$\phi(n,\sigma) \equiv (n-1)^2\left[4n-7\right]\sigma^2 + 4(n-1)\left[-n^2+6n-7\right]\sigma + 4\left[-n^2+4n-3\right] \geq 0.$$

It is easily checked that $\phi(2,\sigma) = \sigma^2 + 4\sigma + 4$ and $\phi(3,\sigma) = 20\sigma^2 + 16\sigma$. Hence, if $n \in \{2,3\}$, then merger to monopoly is sustainable for all $\sigma \in (0,1)$. If $n \geq 4$, then $\phi(n,0) < 0$, and $\phi(n,\sigma)$ has a unique positive root, $\widehat{\sigma}(n)$, given by

$$\widehat{\sigma}(n) = \frac{2\left(n^2-6n+7\right) + 2\sqrt{n^4-8n^3+27n^2-44n+28}}{(n-1)(4n-7)}. \tag{17}$$

Note that $\widehat{\sigma}(n) \in (0,1)$ for all $n \geq 4$. That is, if $n \geq 4$, then merger to monopoly can be supported for all $\sigma \in [\widehat{\sigma}(n), 1)$. ■

Proof of Proposition 2. Suppose the assertion is false. Then, there exist an increasing sequence $\{n^k\}_{k=1}^{\infty}$ of numbers of active firms and a sequence of coalition M_i's number of products, $\{m_i^k\}_{k=1}^{\infty}$, such that M_i's market share, γ_i^k, as measured by the relative number of its products, m_i^k/n^k, is bounded from below by γ, i.e. $\gamma_i^k \geq \gamma$ for all k, and such that $\lim_{k\to\infty}\gamma_i^k = \gamma_i^{\infty}$. (Note that it is always possible to find a convergent

[18] Note that expenditure is maximised under merger to monopoly, in which case $\sum_k p_k^* d_k(\mathbf{p}^*) = n/\left[8(1-\sigma+n\sigma)\right]$.

subsequence since $\gamma_i^k \in [\gamma, 1]$.) For this to be an equilibrium, a member of M_i should have no incentive to deviate and form a coalition on its own. Formally,

$$\frac{\left[1-\sigma+n^k(1-\gamma_i^k)\sigma\right]}{\left[2(1-\sigma)+n^k(2-\gamma_i^k)\sigma\right]^2\left[\Psi^k\right]^2} \geq \frac{\left[1-\sigma+n^k(1-1/n^k)\sigma\right]}{\left[2(1-\sigma)+n^k(2-1/n^k)\sigma\right]^2\left[\Psi^k+\Phi_i^k\right]^2},$$

where

$$\Psi^k \equiv 1-\sigma\sum_{j\in Z}\frac{n^k\gamma_j^k}{2(1-\sigma)+n^k(2-\gamma_j^k)\sigma},$$

and

$$\begin{aligned}\Phi_i^k \;\equiv\; & \frac{\sigma n^k\gamma_i^k}{2(1-\sigma)+n^k(2-\gamma_i^k)\sigma}-\frac{\sigma n^k\left[\gamma_i^k-1/n^k\right]}{2(1-\sigma)+n^k(2-\gamma_i^k+1/n^k)\sigma}\\ & -\frac{\sigma}{2(1-\sigma)+n^k(2-1/n^k)\sigma}.\end{aligned}$$

This condition can be rewritten as

$$\left(\frac{1-\sigma+n^k(1-\gamma_i^k)\sigma}{1-\sigma+n^k(1-1/n^k)\sigma}\right)\left(\frac{2(1-\sigma)+n^k(2-1/n^k)\sigma}{2(1-\sigma)+n^k(2-\gamma_i^k)\sigma}\right)^2 \geq \left(\frac{\Psi^k}{\Psi^k+\Phi_i^k}\right)^2. \qquad (18)$$

Observe that $\lim_{k\to\infty}\Phi_i^k = 0$ since $n^k \to \infty$, and $\gamma_i^k \to \gamma_i^\infty$, as $k \to \infty$. Hence, if Ψ^k is bounded away from zero, the right-hand side of equation (18) converges to 1 as $k \to \infty$, whereas the left-hand side converges to $4(1-\gamma_i^\infty)/(2-\gamma_i^\infty)^2 < 1$. That is, if Ψ^k is bounded away from zero, then for n^k sufficiently large, the above inequality can not hold – a contradiction.

If $\lim_{k\to\infty}\Psi^k = 0$, however, the right-hand side of (18) may not converge to 1. Note that this case occurs if and only if there exists a firm j such that $\gamma_j^\infty = 1$, and hence $\gamma_k^\infty = 0$ for all $k \neq j$. Now, if $i \neq j$, the proof is complete. The interesting case is when $i = j$, i.e. $\gamma_i^\infty = 1$, so that the left-hand side of (18) converges to zero. In fact, the right-hand side of the equation converges to zero as well, provided that firm 1 is a monopolist, $\gamma_i^k = 1$, for k sufficiently large. But we already know from corollary 1 that monopoly cannot be sustained in equilibrium for n sufficiently large. It therefore remains to be shown that the right-hand side of equation (18) is bounded away from zero if $\gamma_i^k \leq (n^k-1)/n^k$, and hence if $\Psi^k > 0$, for sufficiently large k. To show this, remark that $\phi(\gamma) \equiv n\gamma/\left[2(1-\sigma)+n(2-\gamma)\sigma\right]$ is increasing and convex in γ. This implies, first, that Ψ^k is decreasing in the industry level of concentration (where a rise in the level of concentration is defined as a transfer of a certain number of products from some firm to a weakly larger one) and, second, that Φ_i^k is increasing in γ_i^k. Let us define

$$\underline{\Psi}^k \equiv 1-\frac{\left(n^k-1\right)\sigma}{2(1-\sigma)+(n^k+1)\sigma}-\frac{\sigma}{2(1-\sigma)+(2n^k-1)\sigma},$$

and

$$\overline{\Phi}_i^k \equiv \frac{\left(n^k-1\right)\sigma}{2(1-\sigma)+(n^k+1)\sigma} - \frac{\left(n^k-2\right)\sigma}{2(1-\sigma)+(n^k+2)\sigma} - \frac{\sigma}{2(1-\sigma)+(2n^k-1)\sigma}.$$

If firm i is the largest firm, and $\gamma_i^k \leq (n^k-1)/n^k$, we thus have $\Psi^k \geq \underline{\Psi}^k > 0$, $0 < \Phi_i^k \leq \overline{\Phi}_i^k$, and hence

$$\frac{\Psi^k}{\Psi^k + \Phi_i^k} \geq \frac{\underline{\Psi}^k}{\underline{\Psi}^k + \overline{\Phi}_i^k}.$$

It is straightforward to check that the right-hand side of this inequality is bounded away from zero. Hence, for k sufficiently large, equation (18) does not hold. ■

Proof of Proposition 7. For stability condition (ii') to hold, we must have $\overline{m} \to \infty$ as $S/\epsilon \to \infty$. Hence, it suffices to show that, for large $\overline{m}$, the market share of any firm is bounded from above by γ. The proof proceeds along the lines of that of Proposition 2.

Suppose the assertion is false. Then, there exist an increasing sequence $\left\{\overline{m}^k\right\}_{k=1}^{\infty}$ of the number of products and a sequence of the number of firm i's products, $\left\{m_i^k\right\}_{k=1}^{\infty}$, such that firm i's market share γ_i^k, as measured by the relative number of its products, $m_i^k/\overline{m}^k$, is bounded from below by γ, i.e. $\gamma_i^k \geq \gamma$, and such that $\lim_{k\to\infty}\gamma_i^k \equiv \gamma_i^{\infty}$. For $\left(\overline{m}^k\boldsymbol{\gamma}^k\right)$ to be sustainable in an equilibrium configuration, we must have

$$\pi\left(\overline{m}^k\gamma_i^k;\overline{m}^k\boldsymbol{\gamma}_{-i}^k\right) \geq \pi\left(1;\overline{m}^k\boldsymbol{\gamma}^k\right),$$

where $\boldsymbol{\gamma}_{-i}^k$ is the vector of market shares of firm i's rivals. Let us reformulate this inequality as

$$\frac{\pi\left(\overline{m}^k\gamma_i^k;\overline{m}^k\boldsymbol{\gamma}_{-i}^k\right)}{\pi\left(1;\overline{m}^k\boldsymbol{\gamma}^k\right)} = \left(\frac{\Psi^k+\Omega^k}{\Psi^k}\right)^2\Theta_i^k \geq 1, \tag{19}$$

where

$$\begin{aligned}
\Psi^k &\equiv 1-\sigma\sum_j \frac{\overline{m}^k\gamma_j^k}{2(1-\sigma)+\overline{m}^k(2-\gamma_j^k)\sigma}, \\
\Omega^k &\equiv -\frac{\sigma}{2(1-\sigma)+(2\overline{m}^k+1)\sigma} \\
&\quad +\sigma\sum_j\left(\frac{\overline{m}^k\gamma_j^k}{2(1-\sigma)+\overline{m}^k(2-\gamma_j^k)\sigma} - \frac{\overline{m}^k\gamma_j^k}{2(1-\sigma)+\overline{m}^k\left(2\left(1+\frac{1}{\overline{m}^k}\right)-\gamma_j^k\right)\sigma}\right),
\end{aligned}$$

and

$$\Theta_i^k \equiv \frac{\left[1-\sigma+\overline{m}^k(1-\gamma_i^k)\sigma\right]\left[1-\sigma+(\overline{m}^k+1)\sigma\right]\left[2(1-\sigma)+(2\overline{m}^k+1)\sigma\right]^2}{\left[1-\sigma+\overline{m}^k\sigma\right]^2\left[2(1-\sigma)+\overline{m}^k(2-\gamma_i^k)\sigma\right]^2}.$$

It is straightforward to check that

$$\lim_{k\to\infty} \Theta_i^k = \frac{4(1-\gamma_i^\infty)}{(2-\gamma_i^\infty)^2} < 1.$$

Note that $\Theta_i^k, \Psi^k \geq 0$, with strict inequalities under all market structures other than monopoly. Let $\lambda(\overline{m}; m_j) \equiv -m_j / [2(1-\sigma) + (2\overline{m} - m_j)\sigma]$. Since $\eta(m_j) \equiv \partial\lambda(\overline{m}; m_j) / \partial\overline{m}$ is convex in m_j, the candidate equilibrium market structure that maximises Ω^k for a given $\overline{m}^k$, is monopoly; that is,

$$\Omega^k \leq -\frac{\sigma}{2(1-\sigma) + (2\overline{m}^k + 1)\sigma} + \left(\frac{\sigma\overline{m}^k}{2(1-\sigma) + \overline{m}^k\sigma} - \frac{\sigma\overline{m}^k}{2(1-\sigma) + \left(\overline{m}^k + 2\right)\sigma} \right).$$

It is easy to see that the right-hand side of this inequality converges to zero as $\overline{m}^k \to \infty$; hence, $\lim_{k\to\infty} \Omega^k = 0$. If Ψ^k does not converge to zero, i.e. if there is no firm j with $\gamma_j^\infty = 1$, we obtain

$$\lim_{k\to\infty} \left(\frac{\Psi^k + \Omega^k}{\Psi^k} \right) \Theta_i^k = \frac{4(1-\gamma_i^\infty)}{(2-\gamma_i^\infty)^2} < 1,$$

which is in contradiction to equation (19).

If $\lim_{k\to\infty} \Psi^k = 0$, however, $\left(\Psi^k + \Omega^k\right)/\Psi^k$ may not converge to 1. This case occurs if and only if there is a firm j such that $\gamma_j^\infty = 1$, and $\gamma_l^\infty = 0$ for all $l \neq j$. Accordingly, suppose $\gamma_i^\infty = 1$. Since we have already shown in the text that monopoly cannot be sustained as an equilibrium configuration in large markets, let us assume that $\gamma_i^k \leq \left(\overline{m}^k - 1\right)/\overline{m}^k$. Under this assumption,

$$\Psi^k \geq \underline{\Psi}^k \equiv 1 - \frac{\left(\overline{m}^k - 1\right)\sigma}{2(1-\sigma) + (\overline{m}^k + 1)\sigma} - \frac{\sigma}{2(1-\sigma) + (2\overline{m}^k - 1)\sigma},$$

and

$$\begin{aligned} \Omega^k \leq \overline{\Omega}^k \quad \equiv \quad & -\frac{\sigma}{2(1-\sigma) + (2\overline{m}^k + 1)\sigma} \\ & +\sigma \left(\frac{1}{2(1-\sigma) + (2\overline{m}^k - 1)\sigma} - \frac{1}{2(1-\sigma) + (2\overline{m}^k + 1)\sigma} \right) \\ & +\sigma \left(\frac{\overline{m}^k - 1}{2(1-\sigma) + \left(\overline{m}^k + 1\right)\sigma} - \frac{\overline{m}^k - 1}{2(1-\sigma) + \left(\overline{m}^k + 3\right)\sigma} \right), \end{aligned}$$

since $\lambda(\overline{m}; m_j)$ is concave in m_j, and $\partial\lambda(\overline{m}; m_j)/\partial\overline{m}$ convex in m_j, respectively. Accordingly,

$$\frac{\Psi^k + \Omega^k}{\Psi^k} \leq \max\left\{ \frac{\underline{\Psi}^k + \overline{\Omega}^k}{\underline{\Psi}^k}, 1 \right\}.$$

It is easily verified that the right-hand is bounded from above. Hence, if $\gamma_i^\infty = 1$, we obtain

$$\lim_{k\to\infty}\left(\frac{\Psi^k+\Omega^k}{\Psi^k}\right)\Theta_i^k = 0.$$

Again, this is in contradiction to equation (19). ■

Appendix B

The aim of this Appendix is to solve for equilibrium in the constrained endogenous sunk cost game, where firms are not allowed to merge. That is, each firms is associated with a single product, indexed by k.

We begin by solving for consumers' demand function. Suppose income is sufficiently large so that $Y > \sum_k p_k x_k$ in equilibrium. Then, if $d_k(\mathbf{p};\mathbf{u}) > 0$, demand per consumer for good k is given by the first-order condition

$$u_k - 2(1-\sigma)d_k(\mathbf{p};\mathbf{u})/u_k - 2\sigma\sum_{l=1}^{n} d_l(\mathbf{p};\mathbf{u})/u_l - p_k u_k = 0.$$

Relabel firms such that $u_k(1-p_k) \geq u_{k+1}(1-p_{k+1})$ for all $k \in \{1,...,n-1\}$. Define the integer $\overline{n}(\mathbf{p};\mathbf{u})$ in the following way. If $p_1 > 0$, then $\overline{n}(\mathbf{p};\mathbf{u}) = 0$; otherwise, let $\overline{n}(\mathbf{p};\mathbf{u})$ be the largest integer v, $v \leq n$, such that

$$(1-\sigma)u_v(1-p_v) + v\sigma\left[u_v(1-p_v) - \frac{1}{v}\sum_{l=1}^{v} u_l(1-p_l)\right] \geq 0.$$

Demand (per consumer) for k can then be written as

$$d_k(\mathbf{p};\mathbf{u}) = \frac{(1-\sigma)u_k(1-p_k) + \overline{n}(\mathbf{p};\mathbf{u})\sigma\left[u_k(1-p_k) - \frac{1}{\overline{n}(\mathbf{p};\mathbf{u})}\sum_{l=1}^{\overline{n}(\mathbf{p};\mathbf{u})} u_l(1-p_l)\right]}{2(1-\sigma)\left[1-\sigma+\overline{n}(\mathbf{p};\mathbf{u})\sigma\right]}\cdot u_k$$

if $k \leq \overline{n}(\mathbf{p};\mathbf{u})$, and $d_k(\mathbf{p};\mathbf{u}) = 0$ otherwise. To simplify the algebraic expressions, let us define $y_k(\mathbf{p};\mathbf{u}) \equiv d_k(\mathbf{p};\mathbf{u})/u_k$, $q_k \equiv p_k u_k$, $\overline{u}_N(\mathbf{q};\mathbf{u}) \equiv (1/\overline{n}(\mathbf{q};\mathbf{u}))\sum_{l=1}^{\overline{n}(\mathbf{q};\mathbf{u})} u_l$, and $\overline{q}_N(\mathbf{q};\mathbf{u}) \equiv (1/\overline{n}(\mathbf{q};\mathbf{u}))\sum_{l=1}^{\overline{n}(\mathbf{q};\mathbf{u})} q_l$. We thus get the following (normalised) demand function for good k, $k \leq \overline{n}(\mathbf{q};\mathbf{u})$,

$$y_k(\mathbf{q};\mathbf{u}) = \frac{(1-\sigma)u_k + \overline{n}(\mathbf{q};\mathbf{u})\sigma\left[u_k - \overline{u}_N(\mathbf{q};\mathbf{u})\right] - (1-\sigma)q_k - \overline{n}(\mathbf{q};\mathbf{u})\sigma\left[q_k - \overline{q}_N(\mathbf{q};\mathbf{u})\right]}{2(1-\sigma)\left[1-\sigma+\overline{n}(\mathbf{q};\mathbf{u})\sigma\right]},$$

which is continuous in $\mathbf{q}$ and $\mathbf{u}$.

Solving the game by backward induction, we now consider firms' pricing decisions. We take the vector of qualities, $\mathbf{u}$, as given. Suppose $\mathbf{q}^*$ forms a Nash equilibrium in (normalised) prices. Then, if product k makes positive sales, then any product l with $u_l \geq u_k$ makes positive sales as well. To see this, note that firm l, offering quality $u_l \geq u_k$, can ensure itself positive sales and, hence, positive profits by setting price q_l such that $u_l - q_l = u_k - q_k^*$. Relabel firms in decreasing order of quality, i.e. $u_k \geq u_{k+1}$ for all $k \in \{1, ..., n-1\}$. Suppose there are $\overline{n}$ products with positive sales. Omitting all products with zero sales, it is straightforward to show that the equilibrium price of good k, $k \leq \overline{n}$, is given by[19]

$$q_k^*(\overline{n}) = \frac{(1-\sigma)\left[2(1-\sigma)+(2\overline{n}-1)\sigma\right]u_k + \overline{n}\sigma\left[1-\sigma+(\overline{n}-1)\sigma\right](u_k - \overline{u}_N)}{\left[2(1-\sigma)+(\overline{n}-1)\sigma\right]\left[2(1-\sigma)+(2\overline{n}-1)\sigma\right]},$$

where $\overline{u}_N \equiv (1/\overline{n})\sum_{l=1}^{\overline{n}} u_l$ is the average quality of products with positive sales. Equilibrium output can be computed as

$$y_k^*(\overline{n}) = \frac{\left[1-\sigma+(\overline{n}-1)\sigma\right]}{2(1-\sigma)\left[1-\sigma+\overline{n}\sigma\right]} \cdot q_k^*(\overline{n}).$$

The equilibrium number of products with positive sales, $\overline{n}^*$, is uniquely defined as the maximum integer $\overline{n}$, $\overline{n} \leq n$, such that $q_{\overline{n}}^*(\overline{n}) > 0$. Except for the prices of products with zero sales, the equilibrium is unique.

At the third stage, firms (coalitions) simultaneously decide how much to invest in quality. Due to symmetry, any equilibrium is such that $u_k \in \{0, \overline{u}\}$ for all k. That is, each firm spends the same amount on quality, given that it invests at all. Suppose $\widetilde{n}$, $\widetilde{n} \leq n$, firms invest in quality. The equilibrium quality level, $\overline{u}(\widetilde{n})$, is then implicitly defined by the first-order condition

$$S\frac{\left[1-\sigma+(\widetilde{n}-1)\sigma\right]\left[2+3(\widetilde{n}-2)\sigma+(\widetilde{n}^2-5\widetilde{n}+5)\sigma^2\right]}{\left[1-\sigma+\widetilde{n}\sigma\right]\left[2(1-\sigma)+(2\widetilde{n}-1)\sigma\right]\left[2(1-\sigma)+(\widetilde{n}-1)\sigma\right]^2}\overline{u}(\widetilde{n}) - \beta F_0 \overline{u}^{\beta-1}(\widetilde{n}) = 0,$$

which yields

$$\overline{u}(\widetilde{n}) = \left(\frac{S\left[1-\sigma+(\widetilde{n}-1)\sigma\right]\left[2+3(\widetilde{n}-2)\sigma+(\widetilde{n}^2-5\widetilde{n}+5)\sigma^2\right]}{\beta F_0\left[1-\sigma+\widetilde{n}\sigma\right]\left[2(1-\sigma)+(2\widetilde{n}-1)\sigma\right]\left[2(1-\sigma)+(\widetilde{n}-1)\sigma\right]^2}\right)^{\frac{1}{\beta-2}}. \tag{20}$$

It can then be shown that, for the same number of products with positive quality, firms invest more in quality in the completely fragmented market structure than under monopoly; this is due to the "business stealing effect" of investment, which is internalised

[19] This equation implicitly assumes that there is no product with zero sales that constrains equilibrium.

by a monopolist. The profit (net of investment costs) of a firm with quality $\overline{u}(\widetilde{n})$ is of the form $\Pi^*(\widetilde{n}) = \phi(\widetilde{n};\beta,\sigma,S,F_0)\cdot\gamma(\widetilde{n};\beta,\sigma)$, where $\phi(\widetilde{n};\beta,\sigma,S,F_0) > 0$, and

$$\gamma(\widetilde{n};\beta,\sigma) \equiv \beta(1-\sigma)\left[2(1-\sigma)+(2\widetilde{n}-1)\sigma\right] - 2\left[2+3(\widetilde{n}-2)\sigma+(\widetilde{n}^2-5\widetilde{n}+5)\sigma^2\right].$$

It is easy to verify that $\gamma(1;\beta,\sigma) > 0$ and $\lim_{\widetilde{n}\to\infty}\gamma(\widetilde{n};\beta,\sigma) = -\infty$. Since $\gamma(\widetilde{n};\beta,\sigma)$ is a quadratic function of $\widetilde{n}$, it has a unique root $\widetilde{n}^*$ such that $\partial\gamma(\widetilde{n}^*;\beta,\sigma)/\partial\widetilde{n} < 0$, which is given by

$$\widetilde{n}^* = \frac{1}{2\sigma}\left[(\beta-3)-(\beta-5)\sigma+\sqrt{(\beta-1)^2-2\left(\beta^2-3\beta+3\right)\sigma+\left(\beta^2-4\beta+5\right)\sigma^2}\right].$$

We claim that the (maximum) equilibrium number of firms that invest in quality is given by $\min\{[\widetilde{n}^*], n\}$. To see this, note first that qualities of rival products (with positive sales) are strategic substitutes and that $\overline{u}(\widetilde{n})$ is decreasing in $\widetilde{n}$. Hence, if a firm finds it unprofitable to invest in quality when $[\widetilde{n}^*]$ firms have quality level $\overline{u}([\widetilde{n}^*]+1)$, then no firm will find it profitable when the same number of firms offer quality $\overline{u}([\widetilde{n}^*])$. That is, firms offering zero quality in equilibrium have no incentive to deviate. Firms offering positive quality levels in the candidate equilibrium cannot profitably deviate either, since their quality levels are given by the first-order conditions, which satisfy the second-order conditions. It should be noted, however, that more concentrated equilibria may exist.[20]

We now address the first stage at which firms simultaneously decide whether or not to enter the market. Since $\phi(\widetilde{n};\beta,\sigma,S,F_0)\to\infty$ as $S\to\infty$, it follows from the analysis of the investment stage that the maximum equilibrium number of entering firms, n^*, is given by $[\widetilde{n}^*]$ in the limit as market size S tends to infinity, provided that $\widetilde{n}^*$ is not an integer. If $\widetilde{n}^*$ is an integer, then the limit number of entrants is $\widetilde{n}^*-1$. Hence, the equilibrium number of entering firms is bounded, no matter how large the market.

As market size tends to infinity, the limit number of entrants is one if and only if $\sigma\in(\sigma_1(\beta),1)$; it is equal to n, $n\geq 2$, if and only if $\sigma\in(\sigma_n(\beta),\sigma_{n-1}(\beta)]$, where

$$\sigma_n(\beta) \equiv \frac{(2n-3)\beta-6(n-1)+\sqrt{(2n+1)^2\beta^2-4\left(2n^2+5n+1\right)\beta+4\left(n^2+6n+1\right)}}{2\left[(2n-1)\beta+2\left(n^2-3n+1\right)\right]}.$$

Consider two limit cases. First, if $\beta\to\infty$, then $\sigma_n(\beta)\to 1$, and for no value of σ is the limit number of entrants finite.[21] Second, if $\beta\to 2$, then $\sigma_n(\beta)\to 0$, and for all values

[20] A pathological multiplicity arises if $\widetilde{n}^*$ is an integer. In this case, $\min\{[\widetilde{n}^*]-1, n\}$ can also be sustained in equilibrium.

[21] More precisely, we consider the number of entrants as both S and β tend to infinity in such a way that $S/\beta\to\infty$ and $(S/\beta)^{2/(\beta-2)}\to 1$.

of σ the limit number of entering firms is one. Hence, if σ is sufficiently close to one or β sufficiently close to 2, only one firm will enter the market in the limit as the market becomes large.

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