Estimating Time Demand Elasticities Under Rationing.

Victoria Prowse*

Nuffield College, New Road, Oxford, OX1 1NF, UK.

victoria.prowse@nuffield.ox.ac.uk

Telephone: +44(0) 7761447346

Fax: +44(0) 1865 278621

October 14, 2004

Abstract

A multivariate extension of the standard labour supply model in presented. In the multivariate time allocation model leisure is disaggregated into a number of non market activities including sports, volunteer work and home production. Using data from the 2000 UK Time Use Survey, a linear expenditure system is estimated, allowing corner solutions in the time allocated to market work and non market activities. The effects of children, age, gender and education are largely as expected. The unusually high wage elasticities are attributed to a combination of the functional form of the linear expenditure and the treatment of the zero observations.

Key Words: Time use, Labour supply, Corner solutions, Simulation inference.

JEL Classification: C15, C34, J22.

*I would like to thank Steve Bond, Valérie Lechene, Neil Shephard and participants at an Institute for Fiscal Studies seminar, March 2004. This work has been supported by scholarships from the *Journal of Applied Econometrics* and the E.S.R.C, grant number PAT-030-2003-00229.

1 Introduction

In the standard labour supply model all non market time is aggregated into a single quantity, leisure. In this paper, a multivariate extension of the standard labour supply model is presented. In the multivariate time allocation model leisure is disaggregated into a number of different non market activities including sports, volunteer work and home production.

When motivating the model it is useful to review the data. The data is taken from the 2000 UK Time Use Survey which is described in detail below. Table 1 summarises the time use data for males and females. In this table and those below the variable Part. is the proportion of individuals who allocate positive time to the activity, the variable All refers to all individuals and the variable Positive refers to those individuals who allocate a positive amount of time to the activity. Table 1 shows women allocate more time than men to social activities, home production and sleep, whereas men spend longer than women in market work, sports and media activities. Women and men spend a comparable amount of time in volunteer work. The difference in the time allocated to home production activities by males and females is particularly striking. Women spend an average of 30.01 hours per week doing home production activities as compared to 15.12 hours per week by men. There are three non market activities with a sizable proportion of zero observations for both men and women; sports, volunteer work and social activities. Additionally, only 63% of females and 86% of males are observed to spend a positive amount of time in market work.

<Table 1 about here>

Clearly, any reasonable time allocation model must provide an explanation for the zero observations. In the model presented below the zero observations in market work, sports, volunteer work and social activities are treated as corner solutions in individuals' optimisation problems. The zero observations therefore correspond to censored observations. The resulting empirical implementation takes the form of a multivariate Tobit model with endogenous switching, an extension of the model developed in Tobin (1958).

It has long been recognised that when estimating labour supply functions it is important to take account of the censoring in observed hours of market work; ignoring the censoring in observed hours leads to biased and inconsistent estimates of the parameters of the labour supply function (see Wales and Woodland, 1980). In the simplest case, censoring occurs when desired hours of market work are observed only for individuals whose desired hours are positive. For individuals whose desired, or latent, hours of market work are below zero, observed hours of market work are zero. More generally, an individual's desired hours of market work are observed only when their market wage exceeds their reservation wage. In some formulations, the reservation wage is the wage at which desired hours of market work are zero, resulting in the same selection rule as above. However, in the presence of fixed costs or search costs an individual's reservation wage will exceed the wage at which their desired hours of market work are zero.

Likewise, when estimating the multivariate time allocation model it is important to account for censoring in observed hours of market work. Analogously to the treatment of an observation of zero hours of market work in the standard labour supply model, in the multivariate time allocation model an observation of zero hours of market work corresponds to the individual's reservation wage exceeding their market wage. However, when estimating the multivariate time allocation model it is also necessary to account for censoring in the observed time allocated to each non market activity. An observation of zero time allocated to a non market activity corresponds to the individual's virtual price of time in that activity being below their value of time, where their value of time is their wage if they work or their reservation wage if they do not work. As in the case of market work, ignoring censoring in the observed times spent in non market activities leads to biased and inconsistent estimates of the parameters of the model. Consequently, estimates of marginal effects and elasticities will be misleading.

The multivariate time allocation model is implemented by assuming preferences take the Stone Geary form leading to a linear expenditure system for the demand functions. The results provide estimates of the wage elasticity of labour supply and of time in each of the non market activities. The results also give a description of the determinants of the time allocated to the various non market activities, and allow one to quantify the effects demographic variables such as age, education and children have on labour supply and the allocation of time to non market activities.

It is often noted that models of labour supply where observations of zero hours of market work are treated as corner solutions, thus producing Tobit type models, lead to unrealistically high estimates of the wage and income elasticities (see Cogan, 1981, and Mroz, 1987). The results presented here suggest that this problem might be more severe when corner solutions in the time allocated to non market activities are also incorporated.

This paper is related to both the literature on individual labour supply, surveyed by Blundell and MaCurdy (1999) and Killingsworth and Heckman (1986), amongst others, and the literature on time use data, surveyed by Juster and Stafford (1991). The work presented here extends that of Kooreman and Kapteyn (1987) who estimate a multivariate time allocation model, but do not include corner solutions in the time allocated to non market activities, and Kiker and Mendes de Oliveira (1992) who use time use data to examine the problem of selectivity in observed wages. The model is also similar to models used to explain the observed corner solutions in demand data, for example, Lee and Pitt (1986) and Wales and Woodland (1983).

This paper proceeds as follows. Section 2 introduces the multivariate time allocation model. Section 3 presents an empirical implementation of the multivariate time allocation model using the linear expenditure system, gives formulas for reservation wage, virtual prices and wage and non labour market income elasticities and discusses the implications of corner solutions. Section 4 reviews the data. Section 5 presents the results and Section 6 concludes.

2 The Multivariate Time Allocation Problem

Each individual's non market time is disaggregated into m possible uses, denoted by the vector $T_i = [T_{i1}, ..., T_{im}]$ where T_{ij} is the time individual i spends in non market activity j for $i = 1, ..., n$ and $j = 1, ..., m$. Each individual is assumed to have a well behaved utility function, $U(T_i, q_i)$, defined over the time spent in each of the m non market activities and their consumption of the aggregate good, q_i. One may interpret the time spent in non market activities as contributing, via a household production function, to the production of commodities that yield utility, as in Becker (1965). In this case $U(T_i, q_i)$ compounds preferences and technology. With sufficiently strong restrictions on preferences over commodities and on the household technology the utility function $U(T_i, q_i)$ is indeed well behaved (see Pollak and Wachter, 1975).

Individual i faces the following optimisation problem

$$Max_{T_i,q_i} U(T_i, q_i) \tag{1}$$

subject to

$$q_i + w_i \sum_{j=1}^{m} T_{ij} \leqslant w_i T + a_i, \tag{2}$$

$$T_{ij} \geqslant 0, \text{ for } j = 1, ..., m, \tag{3}$$

$$T - \sum_{j=1}^{m} T_{ij} \geqslant 0. \tag{4}$$

Here, $T_{iw} = T - \sum_{j=1}^{m} T_{ij}$ is the time individual i allocates to market work, (2) is the budget constraint while (3) and (4) are non negativity constraints on the time spent in non market activities and market work respectively. The price of the aggregate good has been normalised to one. The complete problem would also include the constraints $T_{ij} \leqslant T$ for $j = 1, ..., m$, and $T_{iw} \leqslant T$, however these constraints are not empirically important and are ignored for what follows. The Kuhn-Tucker conditions for this problem are as follows

$$U_{T_{ij}} - \lambda_i w_i + \mu_{ij} - \eta_i = 0, \text{ for } j = 1, ..., m, \tag{5}$$

$$U_{q_i} - \lambda_i = 0, \tag{6}$$

$$\lambda_i \geqslant 0, \tag{7}$$

$$\mu_{ij} \geqslant 0, \text{ for } j = 1, ..., m, \tag{8}$$

$$\eta_i \geqslant 0, \tag{9}$$

where λ_i is the multiplier on the budget constraint, μ_{ij} is the multiplier on the j^{th} non negativity constraint in (3) and η_i is the multiplier on the non negativity constraint on market work, (4). Subscripts denote partial derivatives. Assuming local non satiation, the budget constraint is strictly binding, implying $\lambda_i > 0$. This allows the first order conditions given by equations (5) to be rearranged to produce

$$U_{T_{ij}} - \lambda_i(\underbrace{\underbrace{w_i + \frac{\eta_i}{\lambda_i}}_{w_i^*} - \frac{\mu_{ij}}{\lambda_i}}_{w_{ij}^*}) = 0, \text{ for } j = 1, ..., m, \tag{10}$$

where w_i^* is individual i's reservation wage and w_{ij}^* is individual i's virtual price of time in non market activity j. Solving the Kuhn Tucker conditions, (5)-(9), produces a system of constrained Marshallian demand functions. Using the definitions of the reservation wage and the virtual prices of time in the constrained non market activities, the constrained demand functions can be expressed as the unconstrained demand functions evaluated at the reservation wage and the virtual prices of the constrained activities, Neary and Roberts (1980). Thus, the demand functions can be written as follows

$$T_j^{mc}(w_i, w_iT + a_i) = T_j^m(w_{i1}^*, ..., w_{im}^*, w_i^*, w_i^*T + a_i), \text{ for } j = 1, ..., m, \tag{11}$$

$$q^{mc}(w_i, w_iT + a_i) = q^m(w_{i1}^*, ..., w_{im}^*, w_i^*, w_i^*T + a_i), \tag{12}$$

where T_j^m and q^m are individual i's *unconstrained* Marshallian demand functions for time in non market activity j and the aggregate good and T_j^{mc} and q^{mc} are individual i's *constrained* Marshallian demand functions for time in non market activity j and the aggregate good.

Intuitively, when an individual drops out of the labour market their value of time is their reservation wage, not their market wage. The individual's decision to allocate time to a non market activity depends on their value of time in the non market activity relative to their reservation wage. Therefore, if an individual allocates zero time to a non market activity while not working in the market it must be that their value of time in the non market activity is less than their reservation wage, which must exceed their market wage.

The primary benefit from expressing the problem in terms of virtual prices arises when deriving comparative statics results. Neary and Roberts (1980) show price and income responses for the demand functions arising as the solutions to the constrained problem can be expressed in terms of the unconstrained demand functions evaluated at virtual prices.

Furthermore, expressing the demand functions in terms of virtual prices makes it clear that an individual's demand for time in unconstrained activities depends on the combination of binding and non binding non negativity constraints facing the individual. An observation of zero time allocated to a non market activity implies a value of time in that activity below the individual's value of time in the activities to which they allocate positive time. This effect, through the virtual price of time in the constrained activity, changes the individual's demand functions for time in the unconstrained activities, relative to the case where the demand for time in the constrained activity is positive. Ignoring any of the corner solutions will lead to a misspecified model. Thus, in order of obtain consistent estimates of the parameters of the model, corner solutions must be explicitly incorporated. This means that if the model is estimated by maximum likelihood, as will be the case below, an individual's contribution to the likelihood depends on the combination of binding and non binding non negativity constraints facing the individual.

It is interesting to note that in the absence of any corner solutions in the time allocated to non market activities it is valid to aggregate the time spent in all non market activities into a single quantity. This is explained as follows. In the absence of any corner solutions in the time allocated to non market activities, an individual's value of time in all non market activities is equal to their wage, if they work, or their reservation wage if they do not work. Thus, the relative prices of the individual's time in all non market activities are fixed, and therefore Hick's (1936) composite commodity theorem can be applied. It follows that aggregation across non market activities is valid and it is possible to correctly estimate the parameters of the labour supply function based on the standard labour supply model.

3 An Empirical Implementation of the Multivariate Time Allocation Model

In this section it is shown that the linear expenditure system can be used to implement the multivariate time allocation model, incorporating corner solutions in the time allocated to non market activities and market work. The model takes the form of a multivariate Tobit with endogenous switching. The utility function and wage equation are specified to include observed and unobserved individual specific heterogeneity. Using the definitions of the reservation wage and virtual prices given above the likelihood can be derived. In addition, closed form expressions can be found for the reservation wage, virtual prices and the wage and non labour market income elasticities of labour supply and of time in non market activities.

When specifying a functional form for preferences it is necessary to choose a utility function that permits corner solutions. Also, given the wage is the price of time in non market activities, the demand functions must not involve cross price effects. For this application preferences are assumed to be of the Stone-Geary form, Stone (1954). This leads to a linear expenditure system for the demand functions. The Stone-Geary utility function takes the following form

$$U(T_i, q_i; \varepsilon_i, Z_i) = \sum_{j=1}^{m} \alpha_{ij} \log(T_{ij} - \gamma_j) + \alpha_{iq} \log(q_i - \gamma_q). \tag{13}$$

The γ_j's can be interpreted as minimum or subsistence quantities. Thus, a corner solution in the time allocated to non market activity j is permitted if γ_j is negative. Such an activity is referred to as inessential.

Maximising (13) subject to the budget constraint, (2), and ignoring the non negativity constraints produces the following system of Marshallian demand functions

$$T_{ij} = \gamma_j + \frac{\alpha_{ij}}{w_i}(w_i T + a_i - w_i \sum_{j=1}^{m} \gamma_j - \gamma_q), \text{ for } j = 1, ..., m, \tag{14}$$

$$q_i = \gamma_q + \alpha_{iq}(w_i T + a_i - w_i \sum_{j=1}^{m} \gamma_j - \gamma_q). \tag{15}$$

Consequently, the labour supply function is given by

$$T_{iw} = \frac{\gamma_q - a}{w_i} + \frac{\alpha_{iq}}{w_i}(w_i T + a_i - w_i \sum_{j=1}^{m} \gamma_j - \gamma_q). \tag{16}$$

Inspecting the above demand functions reveals an absence of cross price effects, as required.

Both observed and unobserved heterogeneity are incorporated into the utility function through the α_i's. The α_i's are specified as follows

$$\alpha_{ij} = \frac{\exp(\varepsilon_{ij} + Z_i'\beta_j)}{\sum_{j=1}^{m} \exp(\varepsilon_{ij} + Z_i'\beta_j) + \exp(\varepsilon_{iq} + Z_i'\beta_q)}, \text{ for } j = 1, ..., m-1, \tag{17}$$

$$\alpha_{im} = \frac{1}{\sum_{j=1}^{m} \exp(\varepsilon_{ij} + Z_i'\beta_j) + \exp(\varepsilon_{iq} + Z_i'\beta_q)}, \tag{18}$$

$$\alpha_{iq} = \frac{\exp(\varepsilon_{iq} + Z_i'\beta_q)}{\sum_{j=1}^{m} \exp(\varepsilon_{ij} + Z_i'\beta_j) + \exp(\varepsilon_{iq} + Z_i'\beta_q)}. \tag{19}$$

Here, Z_i is a vector of observed individual characteristics, and $\varepsilon_i = (\varepsilon_{i1,...,}\varepsilon_{im-1}, \varepsilon_{iq})$ is an m dimensional vector representing the unobserved component of individuals' preferences. The identifying normalisations $\varepsilon_{im} = 0$ for all i and $\beta_m = 0$ have been made. Therefore ε_{ij}, for $j = 1,..,m-1$, represents the unobserved component of individual i's preference for time in non market activity j relative to time in the m^{th} non market activity. Likewise, $Z_i'\beta_j$ represents the observed component of individual i's preference for time in non maket activity j relative to time in the m^{th} non market activitiy. It is assumed that ε_i is known to the individual when they make their time allocation decision, however ε_i is not observed by the econometrician. Furthermore ε_i is assumed to independent of Z_i for $i = 1, ..., n$ and independent across individuals.

In this specification of the linear expenditure system the γ_j's and γ_q are assumed to be constant across individuals. Obviously this is not entirely realistic, for example, one might expect the minimum quantity of goods, γ_q, to vary with the number of children in the household. However, given the already complex nature of the model, incorporating demographic variables in the γ_j's or γ_q is not attempted.

The properties of the above specification of the linear expenditure system are now discussed. The specification of the α_i's given in equations (17)-(19) ensures $0 < \alpha_{ij} < 1$ for $j = 1, ..., m$, $0 < \alpha_{iq} < 1$ and $\sum_{j=1}^{m} \alpha_{ij} + \alpha_{iq} = 1$. The first two conditions are necessary and sufficient for global concavity of the cost function, and therefore ensures negativity. The third condition is necessary and sufficient for the demand functions to satisfy adding up and homogeneity of degree zero in prices and income.

Since the model consists of a system of censored demand functions it is important to ensure the model is coherent (see Gourieroux *et al.*, 1980, Ransom, 1987, van Soest *et al.*, 1993). For the model in hand, coherency requires each realisation of the random variables ε_i to correspond to a unique vector of endogenous variables (T_i, q_i), and for every observed (T_i, q_i) there must exits some ε_i that can generate this outcome. Global concavity of the cost function is sufficient, although not necessary, to ensure the system of censored demand functions is coherent. Since the above stochastic specification ensures negativity is satisfied, the system of censored demand functions is indeed coherent. This allows the model to be estimated without needing to further restrict the parameter space to ensure coherency.

The wage equation is assumed to take the form of log wages being linear in a vector of observable individual characteristics, X_i, with an additive error term, ε_{iw}.

$$\log(w_i) = X_i'\delta + \varepsilon_{iw}. \tag{20}$$

All the error terms are assumed to be identically and independently normally distributed with an unrestricted covariance matrix.

$$\begin{pmatrix} \varepsilon_{iw} \\ \varepsilon_{i1} \\ . \\ \varepsilon_{im-1} \\ \varepsilon_{iq} \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ . \\ . \\ . \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_w^2 & \sigma_{1w} & . & \sigma_{m-1w} & \sigma_{qw} \\ \sigma_{1w} & . & & & . \\ . & & . & & . \\ \sigma_{m-1w} & & & . & \sigma_{qm-1} \\ \sigma_{qw} & \sigma_{1q} & . & \sigma_{qm-1} & \sigma_q^2 \end{pmatrix}\right), \tag{21}$$

where σ_h^2 is the variance of ε_{ih} and σ_{hk} is the covariance between ε_{ih} and ε_{ik}. Correlations between ε_{ij} for $j = 1, ..., m-1$ and ε_{iw} can be attributed to unobserved elements of preferences that affect both individuals' market wages and their demand for time in non market activities. Similarly, correlations between ε_{ih} and ε_{ik} for $h, k = 1, ..., m-1$ and $h \neq k$ can be interpreted as correlations in individuals' unobserved preference for time in the respective non market activities.

The specification of linear expenditure system given above together with the wage equation given by (20) and the stochastic specification given by (21) can be combined to yield explicit expressions for each term in the likelihood. Each individual falls into one of three cases depending on the combination of binding and non binding constraints they are facing. In case (i) all non negativity constraints are non binding, in case (ii) there are binding non negativity constraints on the time allocated to the first l non market activities, and in case (iii) there are binding non negativity constraints on the time allocated to the first l non market activities and also on the time spent in market work.

Each of the three cases are considered in turn. Firstly, consider the first order conditions for case (i), where all non negativity constraints are non binding

$$\frac{\alpha_{ij}}{T_{ij}-\gamma_j}-\lambda_i w_i=0,\ \text{for } j=1,...,m, \tag{22}$$
$$\frac{\alpha_{iq}}{q_i-\gamma_q}-\lambda_i=0. \tag{23}$$

Assume the m^{th} good is always consumed. Dividing the above equations by the m^{th} first order condition and taking logs gives

$$\varepsilon_{ij}=\log(T_{ij}-\gamma_j)-\log(T_{im}-\gamma_m)-Z_i'\beta_j,\ \text{for } j=1,...,m-1, \tag{24}$$
$$\varepsilon_{iq}=\log(q_i-\gamma_q)-\log(T_{im}-\gamma_m)-\log(w_i)-Z_i'\beta_q.$$

Thus, the contribution to the likelihood of an individual who falls into case (i) is given by

$$L_{i1}=f_1(w_i,T_{i1,...,}T_{im-1},q_i|X_i,Z_i) \tag{25}$$
$$=f_{1a}(\varepsilon_{iw},\varepsilon_{i1},...,\varepsilon_{im-1},\varepsilon_{iq}|X_i,Z_i)\left|\frac{\partial\bar{\varepsilon}_i}{\partial\bar{T}_i}\right|, \tag{26}$$

where f_1 is the joint density of $\bar{T}_i=[w_i,T_{i1,...,}T_{im-1},q_i]$ conditional on the observed regressors X_i and Z_i, f_{1a} is the multivariate normal density function of $\bar{\varepsilon}_i=[\varepsilon_{iw},\varepsilon_{i1},...,\varepsilon_{im-1},\varepsilon_{iq}]$ and $\left|\frac{\partial\bar{\varepsilon}_i}{\partial\bar{T}_i}\right|$ is the absolute Jacobian from $\bar{T}_i$ to $\bar{\varepsilon}_i$. Using the budget constraint (2) and the utility function (13) gives

$$\left|\frac{\partial\bar{\varepsilon}_i}{\partial\bar{T}_i}\right|=\begin{vmatrix}\frac{1}{T_{i1}-\gamma_1}+\frac{1}{w_i(T_{im}-\gamma_m)} & . & . & \frac{1}{w_i(T_{im}-\gamma_m)}\\ . & . & & .\\ . & & \frac{1}{T_{im-1}-\gamma_{m-1}}+\frac{1}{w_i(T_{im}-\gamma_m)} & .\\ \frac{1}{w_i(T_{im}-\gamma_m)} & . & . & \frac{1}{q_i-\gamma_q}+\frac{1}{w_i(T_{im}-\gamma_m)}\end{vmatrix}. \tag{27}$$

Moving to case (ii), where the non negativity constraints on the time spent in the first l non market activities are binding, the first order conditions are given by

$$\frac{\alpha_{ij}}{-\gamma_j}-\lambda_i w_{ij}^*=0,\ \text{for } j=1,...,l, \tag{28}$$
$$\frac{\alpha_{ij}}{T_{ij}-\gamma_j}-\lambda_i w_i=0,\ \text{for } j=l+1,...,m, \tag{29}$$
$$\frac{\alpha_{iq}}{q_i-\gamma_q}-\lambda_i=0. \tag{30}$$

Again, dividing by the m^{th} first order condition and taking logs gives

$$P(w_{ij}^* \leqslant w_i|Z_i) = P(\varepsilon_{ij} \leqslant \log(-\gamma_j) - \log(T_{im} - \gamma_m) - Z_i'\beta_j), \text{ for } j = 1, ..., l, \tag{31}$$

$$\varepsilon_{ij} = \log(T_{ij} - \gamma_j) - \log(T_{im} - \gamma_m) - Z_i'\beta_j, \text{ for } j = l+1, ..., m-1, \tag{32}$$

$$\varepsilon_{iq} = \log(q_i - \gamma_q) - \log(w_i) - \log(T_{im} - \gamma_m) - Z_i'\beta_q. \tag{33}$$

Thus, the contribution to the likelihood of an individual who falls into case (ii) is given by

$$L_{i2} = P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i, T_{il+1}, ..., T_{im-1}, q_i, w_i|X_i, Z_i) \tag{34}$$

$$= P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i|T_{il+1}, ..., T_{im-1}, q_i, w_i, X_i, Z_i) f_2(T_{il+1}, ..., T_{im-1}, q_i, w_i|X_i, Z_i) \tag{35}$$

$$= P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i|T_{il+1}, ..., T_{im-1}, q_i, w_i, X_i, Z_i) f_{2a}(\varepsilon_{iw}, \varepsilon_{il+1}, ..., \varepsilon_{im-1}, \varepsilon_{iq}|X_i, Z_i) \left|\frac{\partial \breve{\varepsilon}_i}{\partial \check{T}_i}\right| \tag{36}$$

where $P(.)$ is a l variate normal distribution function, f_2 is the joint density of $\check{T}_i = [w_i, T_{il+1,....,}T_{im-1}, q_i]$ conditional on the observed regressors X_i and Z_i, f_{2a} is the multivariate normal density function of $\breve{\varepsilon}_i = [\varepsilon_{iw}, \varepsilon_{il+1}, ..., \varepsilon_{im-1}, \varepsilon_{iq}]$ and $\left|\frac{\partial \breve{\varepsilon}_i}{\partial \check{T}\,i}\right|$ is the absolute Jacobian from $\check{T}_i$ to $\breve{\varepsilon}_i$. $\left|\frac{\partial \breve{\varepsilon}_i}{\partial \check{T}_i}\right|$ has a similar structure to (27).

In case (iii) the individual faces binding constraints on the time spent in the first l non market activities and also on the time spent in market work. In this case the first order conditions are given by

$$\frac{\alpha_{ij}}{-\gamma_j} - \lambda_i w_{ij}^* = 0, \text{ for } j = 1, ..., l, \tag{37}$$

$$\frac{\alpha_{ij}}{T_{ij} - \gamma_j} - \lambda_i w_i^* = 0, \text{ for } j = l+1, ..., m, \tag{38}$$

$$\frac{\alpha_{iq}}{a_i - \gamma_q} - \lambda_i = 0. \tag{39}$$

Note that in equation (38) the market wage, w_i, has been replaced by the reservation wage, w_i^*. Dividing by the m^{th} first order condition and taking logs gives

$$P(w_{ij}^* \leqslant w_i^*) = P(\varepsilon_{ij} \leqslant \log(-\gamma_j) - \log(T_{im} - \gamma_m) - Z_i'\beta_j), \text{ for } j = 1, ..., l, \tag{40}$$

$$\varepsilon_{ij} = \log(T_{ij} - \gamma_j) - \log(T_{im} - \gamma_m) - Z_i'\beta_j, \text{ for } j = l+1, ..., m-1, \tag{41}$$

$$P(w_i^* \geqslant w_i) = P(\varepsilon_{iq} \leqslant \log(q_i - \gamma_q) - \log(T_{im} - \gamma_m) - \log(w_i) - Z_i'\beta_q). \tag{42}$$

Thus, the contribution to the likelihood of an individual who falls into case (iii) is given by

$$
\begin{aligned}
L_{i3} &= P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i, w_i^* \geqslant w_i, T_{il+1}, ..., T_{im-1}|X_i, Z_i) && (43)\\
&= P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i, w_i^* \geqslant w_i|T_{il+1}, ..., T_{im-1}, X_i, Z_i) f_3(T_{il+1}, ..., T_{im-1}|X_i, Z_i) && (44)\\
&= P(w_{i1}^* \leqslant w_i, ..., w_{il}^* \leqslant w_i, w_i^* \geqslant w_i|T_{il+1}, ..., T_{im-1}, X_i, Z_i) f_{3a}(\varepsilon_{il+1}, ..., \varepsilon_{im-1}|X_i, Z_i) \left| \frac{\partial \tilde{\varepsilon}_i}{\partial \tilde{T}_i} \right| && (45)
\end{aligned}
$$

where $P(.)$ is a $l+1$ variate normal distribution function, f_3 is the joint density of $\tilde{T}_i = [T_{l+1i,....,}T_{m-1i}]$ conditional on the observed regressors X_i and Z_i, f_{2a} is the multivariate normal density function of $\tilde{\varepsilon}_i = [\varepsilon_{il+1}, ..., \varepsilon_{im-1}]$ and $\left|\frac{\partial \tilde{\varepsilon}_i}{\partial \tilde{T}_i}\right|$ is the absolute Jacobian from $\tilde{T}_i$ to $\tilde{\varepsilon}_i$. Again $\left|\frac{\partial \tilde{\varepsilon}_i}{\partial \tilde{T}_i}\right|$ has a similar structure to (27). Combining the probabilities given by (25), (36) and (45) the likelihood can be formed.

When there are individuals facing multiple binding non negativity constraints the likelihood contains high dimensional integrals. The dimension of the integral an individual contributes to the likelihood is equal to the number of binding non negativity constraints facing the individual. Except in special cases, it is computationally difficult to numerically evaluate multivariate normal distribution functions with more than three dimensions. The solution proposed here is to use the GHK simulator due to Börsh-Saupan and Hajivassiliou (1993), Hajivassiliou and McFadden (1990) and Keane (1994) to evaluate the probability each individual contributes to the likelihood.

Briefly, the GHK simulator works as follows. Suppose one wants to find $P(U \leqslant \mu)$ where $U \backsim N(0, \Omega)$, μ is a d dimensional vector and Ω is a d by d covariance matrix. For high dimensions it is computationally difficult to evaluate $P(U \leqslant \mu)$. However, it is possible to find $P(U \leqslant \mu)$ by simulation as follows. Firstly, note that since Ω is positive definite it is possible to find a lower triangular matrix L such that $LL' = \Omega$. Denote the $(i,j)^{\text{th}}$ element of L by L_{ij}. Therefore $L\eta \backsim N(0, \Omega)$ where $\eta \backsim N(0, I_d)$ and I_d is a d dimensional identity matrix. The probability of interest can be approximated as follows

$$
P(U \leqslant \mu) \backsimeq \frac{1}{R} \sum_{r=1}^{R} \Phi\left(\frac{\mu_1}{L_{11}}\right) \Phi\left(\frac{\mu_2 - L_{12}\eta_{2,r}}{L_{22}}\right) ...\Phi\left(\frac{\mu_d - L_{1d}\eta_{1r} - ... - L_{d-1}\eta_{d-1,r}}{L_{dd}}\right), \quad (46)
$$

where $r = 1, ..., R$ indexes the replication and μ_k, $k = 1, ...d$ is the k^{th} element of μ. $\eta_{1,r}$ is the r^{th} draw from a standard normal distribution, $\eta_{2,r}$ is the r^{th} draw from a standard normal distribution truncated from above at $\left(\frac{\mu_1}{L_{11}}\right)$ and so forth.

Börsh-Saupan and Hajivassiliou (1993) shown the GHK simulator generates simulated probabilities that are a continuous and differentiable function of the parameters of the model. This facilitates use of the simulator in maximum likelihood estimation. The authors also show the GHK simulator method produces probability estimates with substantially smaller variance than those generated by acceptance-rejection methods or by Stern's (1992) method. These properties make the GHK simulator an attractive choice for implementing the model in hand.

Using the GHK simulator, the simulated likelihood can be evaluated and and then maximised in the usual way. If the number of replications $R \to \infty$ as the sample size $n \to \infty$, the maximum simulated likelihood estimates are consistent. If $\frac{R}{\sqrt{n}} \to \infty$ as $R \to \infty$ and $n \to \infty$ the maximum simulated likelihood estimates are asymptotically equivalent to the maximum likelihood estimates. Assuming the latter condition is satisfied, all the usual asymptotic likelihood theory applies.

3.1 Elasticities and Virtual Prices

Given the functional form of the linear expenditure system, it is possible to find closed form expressions for the reservation wage and the virtual prices of time in the constrained non market activities. An individual who works in the market and allocates zero time non market activities $j = 1, ..., l$ has a virtual prices of time in the first l non market activities given by

$$w_{ij}^{*} = -\frac{\alpha_{ij}(w_i T + a_i - w_i \sum_{j=l+1}^{m} \gamma_j - \gamma_q)}{\gamma_j(1 - \sum_{j=1}^{l} \alpha_{ij})}, \text{ for } j = 1, ..., l. \tag{47}$$

An individual who does not work in the market and allocates zero time to non market activities $j = 1, ..., l$ and positive time to all other non market activities as a reservation wage and virtual prices for time in the first l non market activities given by

$$w_i^{*} = \frac{(1 - \alpha_{iq})(a_i - \gamma_{iq} - \sum_{j=1}^{l} w_{ij}^{*} \gamma_j)}{\left(T - \sum_{j=l+1}^{m} \gamma_j\right) \alpha_{iq}}, \tag{48}$$

$$w_{ij}^{*} = -\frac{\alpha_{ij}(a_i - \gamma_q)}{\gamma_j \alpha_{iq}}, \text{ for } j = 1..., l. \tag{49}$$

The wage and non labour market income elasticities of labour supply and of time in non market activities can be found by combining the expressions for the demand functions given in equations (14)-(16) and the formulas for the reservation wage and virtual prices given in equations (47)-(49). Below, the formulas for the wage elasticities of labour supply and of time in non market activities are presented, for the case where there are binding constraints on the time spent in the first l non market activities.

$$\varepsilon_{w,T_w} = \frac{(1-\alpha_{iq}-\sum_{j=1}^{l}\alpha_{ij})(a_i-\gamma_q)}{T_{iw}w(1-\sum_{j=1}^{l}\alpha_{ij})}, \tag{50}$$

$$\varepsilon_{w,T_j} = -\frac{\alpha_{ij}(a_i-\gamma_q)}{T_{ij}w_i(1-\sum_{j=1}^{l}\alpha_{ij})}, \text{ for } j=l+1,...,m. \tag{51}$$

Similarly, the non labour market income elasticities of labour supply and of time in non market activities for the case where there are binding constraints on the time spent in the first l non market activities are given by

$$\varepsilon_{a,T_a} = -\frac{(1-\alpha_{iq}-\sum_{j=1}^{l}\alpha_{ij})a_i}{T_{iw}w(1-\sum_{j=1}^{l}\alpha_{ij})}, \tag{52}$$

$$\varepsilon_{a,T_j} = \frac{\alpha_{ij}a_i}{T_{ij}w_i(1-\sum_{j=1}^{l}\alpha_{ij})}, \text{ for } j=l+1,...,m. \tag{53}$$

The wage elasticities of labour supply and of time in non market activities are zero for individuals who do not work in the market. Also, the non labour market income elasticity of labour supply is zero for individuals who do not work in the market.

Inspecting the above formulas reveals that time in each non market activity is a substitute for time in market work. The above formulas also show the wage elasticities depend on the parameters γ_q, α_{ij} for $j=1,...,m$ and α_{iq}, whereas the non labour market income elasticities depend only on α_{ij} for $j=1,...,m$ and α_{iq} . Thus, given the restriction $\sum_{j=1}^{m}\alpha_{ij}+\alpha_{iq}=1$, the total effect of demographic variables and non labour market income on the demand functions and elasticities is restricted.

3.2 Corner Solutions and the Linear Expenditure System

The effects of treating the zero observations in market work and non market activities as corner solutions in individuals' optimisation problems are now considered. This is first discussed for the general case then specialised to the linear expenditure system.

As noted above, researchers modelling labour supply with Tobit models often comment on the unrealistically high wage and income effects implied by these models (see Cogan, 1981 and Mroz, 1987, for example). When labour supply is modelled within the Tobit framework high wage and income effects arise as the probability of non participation is closely tied to the wage and income effects. This occurs as, in the Tobit model, an individual does not participate in market work if their latent supply of time to market work is negative. Thus, in order to predict some individuals at corner solutions with respect to market work and positive hours of market work for other individuals the range of latent predicted demands must be greater than when only positive hours of market work are predicted. This requires either a greater wage effect or a larger income effect. Alternatively, the required variation in latent predicted demands can be achieved by a greater effect of demographic variables in the labour supply function.

Extending this logic to the multivariate time allocation problem suggests that when the zero observations in non market activities are treated as corner solutions the effects of the wage or non labour market income or the effects of demographic variables on the time allocated to there activities will also be greater than in the absence of corner solutions. The consequent effect on labour supply will depend on how individuals reallocate their time between activities in response to changes in their wage, non labour market income or demographic characteristics.

In the case of the linear expenditure system, allowing corner solutions in market work and non market activities will tend to increase the estimated wage elasticities. Intuitively, the effects non labour market income and demographic variables on the time allocation decision are limited by the restriction $\sum_{j=1}^{m} \alpha_{ij} + \alpha_{iq} = 1$. Therefore, variation in non labour market income or demographic variables is unlikely to generate a sufficiently large range of latent predicted demands. Instead, the required variation in latent predicted demands is generated by a large wage effect. Furthermore, in the linear expenditure system time in each non market activity is a substitute for time in market work. Therefore, if the time allocated to some non market activities is highly wage sensitive, as is expected if a sizable proportion of individuals are at corner solutions with respect to the time allocated to these non market activities, the time allocated to market work will necessarily be highly wage sensitive. Thus, it is apparent that treating all zero observations as corner solutions and using the linear expenditure system to estimate the multivariate time allocation model is likely to lead to still higher wage elasticities than those found when estimating the standard labour supply model within the Tobit framework.

4 An Overview of the Data

The data is taken from the 2000 UK Time Use survey. The main aim of this survey was to measure how individuals allocate their time between various activities. The primary sampling unit consisted of postcode sectors divided into Government Office Regions. Within these postcode sectors, account was taken of the population density and the social economic group of the head of household. All individuals aged 8 years or over were asked to complete two 24 hour time use diaries, one for a weekday and one for a weekend day. For every 10 minute interval in each 24 period individuals were asked to record primary and secondary activities as well as information on their location and who they were with. Household and individual questionnaires were used gather background information and demographics. All those in work or education were also asked to complete a one week work and education record detailing the time spent in work and full time education over the week in which the time use diaries were completed.

The model is implemented using a sub sample consisting of married or cohabiting adults, and is estimated separately for males and females. The samples consist of 1832 females and 1433 males. Retired individuals and students have been excluded. As in common when using time use data, an equivalent week has been constructed for each individual. The time an individual spends on each activity during an equivalent week is defined as five times the weekday diary observation plus two times the weekend observation. Eight different time uses are distinguished, and the definition of each is given below.

Market work Working time in main job, coffee breaks and other breaks in main job, working time in secondary job, coffee breaks or other breaks in secondary job, other activities relating to employment, excluding activities relating to job search.

Sports Sports and outdoor activities, physical exercise, productive exercise, hobbies and games, computing, collecting, correspondence, solo games, games played with others, computer games and gambling.

Volunteer Work Volunteer work and meetings, work for an organisation, volunteer work through an organisation, other organisational work, informal help to other households, participatory voluntary activities including meetings and religious activities.

Social activities Social life and entertainment, socialising with household members, visiting and receiving visitors, feasts, telephone conversations, cinema, theatre and concerts, art exhibitions and museums, library, sports events, resting, other entertainment and culture.

Home production Food management and preparation, cleaning dwelling, cleaning yard, making and care of textiles, gardening and pet care, house construction and renovation, shopping, commercial or administrative services, personal services, care of another household member, including childcare.

Media activities Reading, watching television, listening to the radio, music or recordings, other mass media activities.

Other time use Classes and lectures, homework, other activities relating to school or university, free time study, travel related to work, activities relating to job search, other unspecified

Sleep Sleep, sick in bed, eating, washing and dressing, other activities relating to personal care.

Table 1 summaries the time use data for primary activities for males and females. In total, there are sixteen different combinations of binding and non binding non negativity constraints. The numbers of individuals facing each combination of binding and non binding non negativity constraints are given in Table 2.

<Tables 2 and 3 about here>

Table 3 summarises the demographic and wage data for males and females. The regressors used in the wage equation are age, age squared, education and an intercept, and Z_{ij} consists of age, age squared, education, the number of children in the household and an intercept. Here, age is age is years, education is an indicator variable taking the value one if the individual has an educational level of A Levels or above and zero otherwise and children is the number of children under 16 years of age present in the household. Wage data for employed and self employed individuals was collected via the individual questionnaire. Employed individuals were asked to report their last take home pay after deductions and the period covered by their last take home pay. Individuals refusing to answer this question were asked to report their monthly take home pay. Self employed individuals were asked to report their monthly take home pay. All working individuals were asked to report the hours worked in a typical week. Using this information the hourly wage, in £, was constructed for all individuals in employment.

Non labour market income is defined as weekly household income less the weekly labour market income of all household members, divided by the number of household members. Consumption of the aggregate good is defined as weekly non labour market income, plus the wage times hours of market work during the equivalent week. Thus, income is assumed to be equal to consumption, and the possibility of consumption smoothing has been excluded. Furthermore, no attempt has been made to allocate household income between household members in a way that reflects the differing needs or consumption of household members. Alternatively, an equivalence scale could be used to adjust each members income according to the composition of the household.

5 Results

When estimating the model the number of replication used when simulating the likelihood has been set at 20. The results appear to be robust to the number of replications. Numerical calculations were performed using MATLAB. Parameter estimates are given in Tables 4 - 8.

<Tables 4 - 8 about here>

Before accessing the predicted elasticities and time allocations it is interesting to discuss some of the estimated parameter values. The parameters of the wage equations are reasonable. The rate of return to gaining a educational level of A Levels or above is 44% for females and 33% for males. For both males and females log wages appear to be quadratic in education. For males the γ_j's are negative for all non market activities and for females they are negative for all non market activities except sleep. The specification implemented here constrains γ_1, γ_2 and γ_3, corresponding to sports, volunteer work and social activities to be negative but places no restrictions on the other γ_j's. However, finding negative γ_j's for the other activities is not inconsistent with the framework presented above; it is possible for an activity to have a negative γ and yet no individual be observed at a corner solution with respect to this activity. The values of γ_q are far lower than the values of the γ_j's. Considering the asymmetric way in which γ_q enters into the model, relative to the γ_j's, this is not surprising. Indeed, given the way in which γ_q enters the formulas for the wage elasticities, this finding suggests the required variation in latent predicted demands is largely being generated by uniformly large wage effects, and not by the effects of demographic variables or non labour market income.

Tables 7 and 8 give the estimated covariance matrices for males and females. Examining these tables shows females' unobserved preference for time in volunteer work is negative correlated with the error in their wage equation. For males, the unobserved preferences for time in both volunteer work and social activities are negatively correlated with the error in their wage equation. For both males and females all other correlations of unobserved preferences with the error in the wage equation are positive. It is interesting to note that the correlation of the unobserved preferences for time in social activities and media activities is negative for both males and females. This means individuals who have a high unobserved taste for time in media activities are centris paribus likely to have a relatively low preference for time in social activities, and vice versa. A similar interpretation can be given to the other correlations in these tables. The parameters relating to demographic variables are most readily interpreted in terms of the effect they have of the wage elasticities and time allocations, see below.

The demand functions given by equations (14)-(16) and the wage elasticities given by equations (50) and (51) depend on both the observed individual specific heterogeneity, Z_{ij}, and the unobserved heterogeneity, ε_i. Since ε_i is unobserved the estimated elasticities and demand functions are evaluated by simulation. To access the fit of the model and obtain wage elasticities for the individuals in the sample the observed Z_{ij} and non labour market income for each individual are used and 100 values of ε_i are drawn for each individual. Table 9 presents the results for this simulation. The columns headed T give the mean predicted time in hours per week allocated to each activity, and the columns headed ε give the median wage elasticity of time in each activity.

<Table 9 about here>

Table 9 shows the model predicts women allocate more time than men to social activities, home production and sleep, whereas men spend longer than women in market work, sports and media activities. The mean predicted time allocated to volunteer work and other time use is similar for men and women. This is entirely consistent with the observed time allocations summarised in Table 1. The mean predicted time spent in market work is 20.33 hours per week for women and 38.29 hours per week for men. This figures compare favorably with the observed hours of market work of 21.04 for women and 38.41 for men. The mean predicted times spent in non market activities for men and women also mirror the observed time allocations.

<Figures 1 and 2 about here>

It is also insightful to look at the distributions of predicted times and compare these to the distributions of observed times. These distributions are illustrated in Figure 1, for females, and Figure 2, for males. For most non market activities the distributions of predicted and observed times show a close resemblance. An exception occurs in the case of the time allocated to home production by males. The distribution of predicted hours has a higher mode and is more skewed to the right than the distribution of observed hours. Turning to market work, the distributions of predicted and observed hours are decidedly different. For women, the distribution of observed hours of market work has a small peak at around 20 hours per week, corresponding to part time employment, and a larger peak at 40 hours per week, corresponding to full time employment. However, the distribution of predicted hours of market work is much flatter and is unimodal. For men, the distribution of observed hours is concentrated at 40 hours per week, however the distribution of predicted hours is again much flatter. These differences are suggestive of there being additional constraints on the time allocated to market work. These constraints may take the form of individuals being unable to freely choose their hours of work, and instead facing a choice between a finite set of alternatives characterised by different hours of market work (for an example in the context of the standard labour supply model see van Soest, 1995).

The model gives reasonable predictions of the proportions of non participants for market work, sports, volunteer work and social activities, although these appear to be somewhat more accurate for males than for females. A by product of predicting high proportions of non participants in market work and non market activities is higher wage elasticities then those commonly found in the labour supply literature. The results suggest a wage elasticity of labour supply of 3.97% for females and 3.72% for males. For working individuals the median wage elasticity of labour supply is 6.45% for females and 4.06% for males. As noted above, this difference is consistent with the functional form of the linear expenditure system and the treatment of the zero observation. Ideally, one would specify a less restrictive functional form, where the probability of non participation is not closely tied to the wage elasticity. This could be achieved by using a functional form which does not restrict all non market activities to be substitutes for market work. However, with more flexible functional forms ensuring coherency of the demand system becomes more difficult (see Diewert and Wales, 1987 and Pitt and Millimet, 2001). An alternative method of breaking the link between participation and the wage elasticity would be to incorporate fixed costs of supplying time to activities, as suggested, in the context of labour supply, by Cogan (1981). This would allow the model to predict non participation without implying a large wage effect. For labour supply, fixed costs might take the form of transport or childcare costs. However, the presence of fixed costs of allocating time to non market activities is less clear.

<Tables 10-12 about here>

Tables 10-12 detail the effects of demographic variables on the probability of participation, the wage elasticities and the allocation of time to market work and non market activities. The results are based on a simulation of 1000 individuals and refer to individuals aged 30 with no children, a low level of education and a non labour market income of £20 per week. The effect of age is similar for males and females. A one year increase in an individual's age increases the time they spend in market work by an average of 0.53 hours per week for females and 0.63 hours per week for males. The probability an individual works in the market in also increasing the individual's age, whereas the wage elasticity of labour supply is decreasing in age. The time spent in sports, social activities, other time use and sleep is decreasing in age for males and females, whereas the time spent in social activities and home production is increasing in age. The time spent in media activities is increasing in age for females and decreasing in age for males. While some of these effects appear large, it should be noted that age appears in the wage equation and the demand functions in a quadratic form so the marginal effect of age depends on where the effect is evaluated. For example, given the estimated parameter values, the effect of age on labour supply much smaller for individuals aged 50 than for individuals aged 30.

Table 11 shows the effect of increasing education from a low level to a high level. For females, education appears to mainly affect the time allocated to market work, sports and media activities. Females with a high level of education spend 1.84 hours per week longer in market work than otherwise identical individuals with a low level of education. They also spend 1.48 hours per week longer in sports activities and 3.35 hours per week less in media activities. In contrast, males with a high level of education spend 1.95 hours per week less in market work than males with a low level of education. When education increases from a low level to a high level, the time males spend in media activities and sleep decreases by 2.20 and 1.02 hours per week respectively, and the time spent in sports and other time use increases by 2.64 and 2.44 hours per week respectively. Females with a high level of education are more likely to participate in market work and social activities whereas males with a high level of education are less likely to participate in these activities. For both males and females, a high level of education increases the probability of participation in sports activities and decreases the probability of participation in volunteer work.

Finally, Table 12 shows the effect of increasing the number of children present in the household from zero to one. For females, the greatest effect of an increase in the number of children is on the time allocated to market work and home production. The presence of a child in the household decreases the time spent in market work by 6.46 hour per week and increases the time spent in home production by 5.38 hours per week. For males, the presence of a child reduces the time spent in market work by 1.76 hours per week and decreases the time spent sleeping by 1.02 hours per week. The time spent in home production increases by 0.97 hours per week. Thus, the presence of a child in the household appears to have a greater effect on the time allocations of women than of men.

6 Conclusion

A multivariate time allocation model with corner solutions in the time allocated to non market activities and market work has been estimated, assuming Stone Geary preferences, which produces a linear expenditure system for the demand functions. Unobserved heterogeneity takes the form of random preference variation and unobserved wage variation. The computation problems posed by the high dimensional integrals in the likelihood have been circumvented by using the GHK simulator.

The model gives reasonable predictions of the proportions individuals who do not participate in market work and non market activities, and of the time allocated to market work and non market activities. However, the estimated wage elasticity of labour supply is higher than that typically found in the labour supply literature. This discrepancy has been attributed to the functional form employed here, together with the high proportion of individuals who are observed to be at a corner solution with respect to the time allocated to market work or non market activities.

An interesting extension of this work would be to attempt to implement the multivariate time allocation model using a more flexible functional form, where it would be possible to accommodate individuals at corner solutions without constraining the wage elasticity to be high. Another extension would be to attempt to incorporate additional constraints on the allocation of time to market work and investigate whether this produces a distribution of predicted hours of market work that more closely resembles the distribution of observed hours of market work.

References

Börsh-Saupan, A. and V. Hajivassiliou (1993), Smooth unbiased multivariate probability simulators for maximum likelihood estimation of limited dependent variable models, *Journal of Econometrics*, **58**(3): 347-68.

Becker, G. (1965), A theory of the allocation of time, *The Economic Journal,* **75**(299): 493-517.

Blundell, R. and T. MaCurdy (1999), Labour supply, in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, Vol 3a, Amsterdam: North Holland.

Cogan, J. (1981), Fixed costs and labor supply, *Econometrica*, **49**(4): 945-963.

Diewert, W. and T. Wales (1987), Flexible functional forms and global curvature conditions, *Econometrica,* **55**(1): 43-68.

Gourieroux, C., J. Laffont and A. Monfort (1980), Coherency conditions in simultaneous linear equation models with endogenous switching regimes, *Econometrica,* **48**(3): 675-696.

Hajivassiliou, V. and D. McFadden (1990), The method of simulated scores for the estimation of LDV-models with an application to external debt crises; *Cowles Foundation Discussion Paper,* Number 967; Yale University.

Hicks, J. (1936), *Value and capital*, Oxford, Oxford University Press.

Ipsos-RSL and Office for National Statistics, (2003), *United Kingdom Time Use Survey, 2000, 3rd Edition*, Colchester, Essex: UK Data Archive.

Juster, F. and F. Stafford (1991), The allocation of time - Empirical findings, behavioral models, and problems of measurement, *Journal of Economic Literature* **29**(2)*:* 471-522.

Keane, M. (1994), A computationally practical simulation estimator for panel data, *Econometrica* **62**(1): 95-116.

Kiker, B. and M. Mendes de Oliveira (1992), Optimal allocation of time and estimation of market wage functions, *The Journal of Human Resources,* **27**(3): 445-471.

Killingsworth, M. and J. Heckman (1986), Female labor Ssupply: a Survey, in Ashenfelter, O. and P. Layard (eds.), *Handbook of Labor Economics*, Vol.1, Amsterdam: North Holland.

Kooreman, P. and A. Kapteyn (1987), A disaggregated analysis of the allocation of time within the household, *The Journal of Political Economy,* **95**(2): 223-249.

Lee, Lung-Fei. and M. Pitt (1986), Microeconometric demand system with binding nonnegativity constraints: The dual approach, *Econometrica,* **54**(5): 1237-1242.

Mroz, T. (1987), The sensitivity of an empirical model of married women's hours of work to economic and statistical assumptions, *Econometrica*, **55**(4): 765-799.

Neary, J. P. and K. Roberts (1980), The theory of household behavior under rationing, *European Economic Review,* **13**(1):25-42.

Pollak, R. and M. Wachter (1975), The relevance of the household production function and its implications for the allocation of time, *The Journal of Political Economy,* **83**(2): 255-278.

Ransom, M. (1987), A comment on consumer demand systems with binding non negativity constraints, *Journal of Econometrics,* **34**(3): 355:359.

Stern, S. (1992), A method for smoothing simulated moments of discrete probabilities in multinomial probit models, *Econometrica*, **60**(4): 943-952.

Stone, J. (1954), Linear expenditure systems and demand analysis: an application to the pattern of British demand, *Economic Journal*, **64**(255): 511-527.

Tobin, J. (1958), Estimation of relationships for limited dependent variables, *Econometrica*, **26**(1): 24-36.

van Soest, A. (1995), Structural models of family labor supply: A discrete choice approach, *The Journal of Human Resources*, **30**(1): 63-88.

van Soest, A., A. Kapteyn and P. Kooreman (1993), Coherency and regularity of demand systems with equality and inequality constraints, *Journal of Econometrics,* **57**(1): 161-188.

Wales, T. and A. Woodland (1980), Sample selectivity and the estimation of labor supply functions, *International Economic Review*, **21**(2): 437-68.

——— (1983), Estimation of consumer demand systems with binding non-negativity constraints, *Journal of Econometrics,* **21**(3): 263-285.

Activity	Females 1832 observations			Males 1433 observations		
	Part	Positive	All	Part	Positive	All
Market work	0.63	33.40	21.04	0.86	45.08	38.41
Sports	0.45	6.12	2.75	0.54	8.45	4.55
Volunteer Work	0.27	7.03	1.89	0.11	6.02	1.24
Social activities	0.91	10.38	9.49	0.86	8.67	7.49
Home production	0.99	30.24	30.01	0.97	15.64	15.12
Media activities	0.97	16.55	16.09	0.98	19.00	18.68
Other time use	0.99	12.70	12.58	0.99	12.49	12.43
Sleep	1	74.15	74.15	1	70.09	70.09

Table 1: Summary of Time Use data: Hours per equivalent week.

Category	Females	Males	Category	Females	Males
W, Sp, V, Sc	118	112	$\underline{W}, \underline{Sp}, V, Sc$	81	20
$W, \underline{Sp}, V, Sc$	158	98	$\underline{W}, Sp, \underline{V}, Sc$	216	84
$W, Sp, \underline{V}, Sc$	336	438	$\underline{W}, Sp, V, \underline{Sc}$	4	1
$W, Sp, V, \underline{Sc}$	6	5	$W, \underline{Sp}, \underline{V}, \underline{Sc}$	65	84
$\underline{W}, Sp, V, Sc$	112	45	$\underline{W}, Sp, \underline{V}, \underline{Sc}$	206	47
$W, \underline{Sp}, \underline{V}, Sc$	448	394	$\underline{W}, \underline{Sp}, V, \underline{Sc}$	4	0
$W, \underline{Sp}, V, \underline{Sc}$	8	13	$\underline{W}, \underline{Sp}, \underline{V}, Sc$	15	10
$W, Sp, \underline{V}, \underline{Sc}$	15	77	$\underline{W}, \underline{Sp}, \underline{V}, \underline{Sc}$	40	5

Table 2: Number of individuals falling into each combination of binding and non binding non negativity constraints: W, Sp, V and Sc denote Market work, Sports, Volunteer work and Social activities respectively. An underscore denotes a zero observation for the corresponding activitiy.

	Females		Males	
	Mean	s.d	Mean	s.d
Age	39.84	11.01	40.50	11.62
Education	0.28	0.45	0.29	0.46
Children	1.22	1.25	1.05	1.16
Wage	6.64	5.05	8.43	5.54

Table 3: Summary of demograpgic and wage data.

	Females	Males
γ_1	-13.25 (0.58)	-10.13 (0.65)
γ_2	-53.04 (1.38)	-52.19 (1.87)
γ_3	-7.22 (0.42)	-5.28 (0.36)
γ_4	-13.57 (0.77)	-3.62 (0.39)
γ_5	-7.71 (0.48)	-12.96 (0.63)
γ_6	-4.78 (0.40)	-3.22 (0.37)
γ_7	4.50 (0.028)	-15.07 (0.83)
γ_q	-38327.45 (886.50)	-43162.57 (909.48)

Table 4: Estimates of γ_j for j=1,...,7 and γ_q for females and males. Standard errors in parenthesis. γ_1,...,γ_7 correspond to sports, volunteer work, social activities, home production, media activities, other time use and sleep respectively.

	Females	Males
Intercept	1.16 (0.088)	1.34 (0.15)
Age	0.021 (0.0048)	0.024 (0.0069)
Age^2	-0.00022 (0.000066)	-0.00023 (0.000083)
Education	0.44 (0.025)	0.33 (0.029)

Table 5: Estimated of parameters of the wage equation for females and males.

	Females	Males		Females	Males
β_1			β_5		
Intercept	-1.94 (0.18)	-2.04 (0.21)	Intercept	-1.34 (0.12)	-1.14 (0.10)
Age	0.0085 (0.0097)	-0.0031 (0.011)	Age	0.0083 (0.0064)	0.0024 (0.0054)
Age^2	-0.000072 (0.00011)	0.000010 (0.00013)	Age^2	-0.000052 (0.000076)	0.000013 (0.0063)
Education	0.13 (0.027)	0.22 (0.039)	Education	-0.13 (0.026)	-0.079 (0.022)
Number of children	-0.019 (0.011)	0.033 (0.017)	Number of children	-0.019 (0.020)	-0.0044 (0.0093)
β_2			β_6		
Intercept	-0.78 (0.086)	-0.98 (0.076)	Intercept	-1.53 (0.16)	-1.83 (0.17)
Age	0.014 (0.0040)	0.011 (0.0041)	Age	0.0046 (0.0085)	-0.0010 (0.0093)
Age^2	-0.011 (0.000047)	-0.000091 (0.000048)	Age^2	-0.00011 (0.000097)	-0.00001 (0.011)
Education	0.000001 (0.016)	0.000006 (0.018)	Education	0.066 (0.026)	0.15 (0.031)
Number of children	0.013 (0.0063)	0.019 (0.0076)	Number of children	0.019 (0.010)	0.028 (0.013)
β_3			β_q		
Intercept	-0.94 (0.16)	-1.55 (0.18)	Intercept	4.73 (0.054)	4.55 (0.15)
Age	-0.033 (0.0077)	-0.030 (0.0090)	Age	-0.0004 (0.0043)	-0.0064 (0.0081)
Age^2	0.00041 (0.000092)	0.00038 (0.00011)	Age^2	-0.000027 (0.0061)	0.000030 (0.000057)
Education	0.023 (0.028)	-0.0018 (0.038)	Education	-0.42 (0.026)	-0.32 (0.030)
Number of children	0.019 (0.011)	0.0013 (0.016)	Number of children	-0.012 (0.0038)	0.0052 (0.0035)
β_4					
Intercept	-1.02 (0.11)	-2.4616 (0.23)			
Age	0.0079 (0.0055)	0.0148 (0.013)			
Age^2	0.000004 (0.000066)	0.000009 (0.014)			
Education	-0.019 (0.022)	0.0135 (0.039)			
Number of children	0.13 (0.0090)	0.1165 (0.017)			

Table 6: Estimates of β_j for j=1,...,6 and β_q for males and females. Standard errors in parenthesis. β_1,...,β_6 correspond to sports, volunteer work, social activities, home production, media activities and other time use respectively.

	ε_w	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_q
ε_w	0.15	0.0024	-0.0016	0.025	0.0083	0.018	0.016	0.0024
ε_1	.	0.22	0.028	0.025	0.0062	0.017	0.047	0.020
ε_2	.	.	0.066	0.039	0.018	0.016	0.045	0.026
ε_3	.	.	.	0.28	0.016	-0.019	0.045	0.025
ε_4	.	.	.	.	0.17	0.018	0.0016	0.016
ε_5	.	.	.	.	.	0.23	-0.017	0.021
ε_6	.	.	.	.	.	.	0.26	0.040
ε_q	.	.	.	.	.	.	.	0.029

Table 7: Females: Estimated covariance matrix.

	ε_w	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	ε_q
ε_w	0.23	0.011	-0.011	-0.0032	0.0096	0.0058	0.030	-0.0028
ε_1	.	0.38	0.014	0.00081	-0.038	0.027	0.0080	0.012
ε_2	.	.	0.057	0.033	0.019	0.0044	0.032	0.017
ε_3	.	.	.	0.41	0.046	-0.026	0.071	0.023
ε_4	.	.	.	.	0.43	0.014	-0.0041	0.018
ε_5	.	.	.	.	.	0.14	-0.0071	0.015
ε_6	.	.	.	.	.	.	0.27	0.025
ε_q	.	.	.	.	.	.	.	0.020

Table 8: Males: Estimated covariance matrix.

Activity	Females					Males				
	Part.	T		ε		Part.	T		ε	
		Positive	All	Positive	All		Positive	All	Positive	All
Market work	0.68	29.67	20.33	6.45	3.97	0.91	42.20	38.29	4.06	3.72
Sports	0.30	6.14	2.79	-4.08	0	0.54	8.31	4.51	-2.76	0
Volunteer Work	0.17	7.05	1.88	-11.04	0	0.20	6.56	1.31	-11.40	0
Social activities	0.62	10.48	9.67	-1.87	-1.44	0.86	8.66	7.45	-1.79	-1.55
Home production	1	30.11		-1.37		1	15.31		-1.23	
Media activities	1	16.30		-1.34		1	18.63		-1.65	
Other time use	1	12.58		-1.25		1	12.35		-1.23	
Sleep	1	74.34		-0.89		1	70.14		-1.17	

Table 9: Mean predicted times and median predicted wage elasticities.

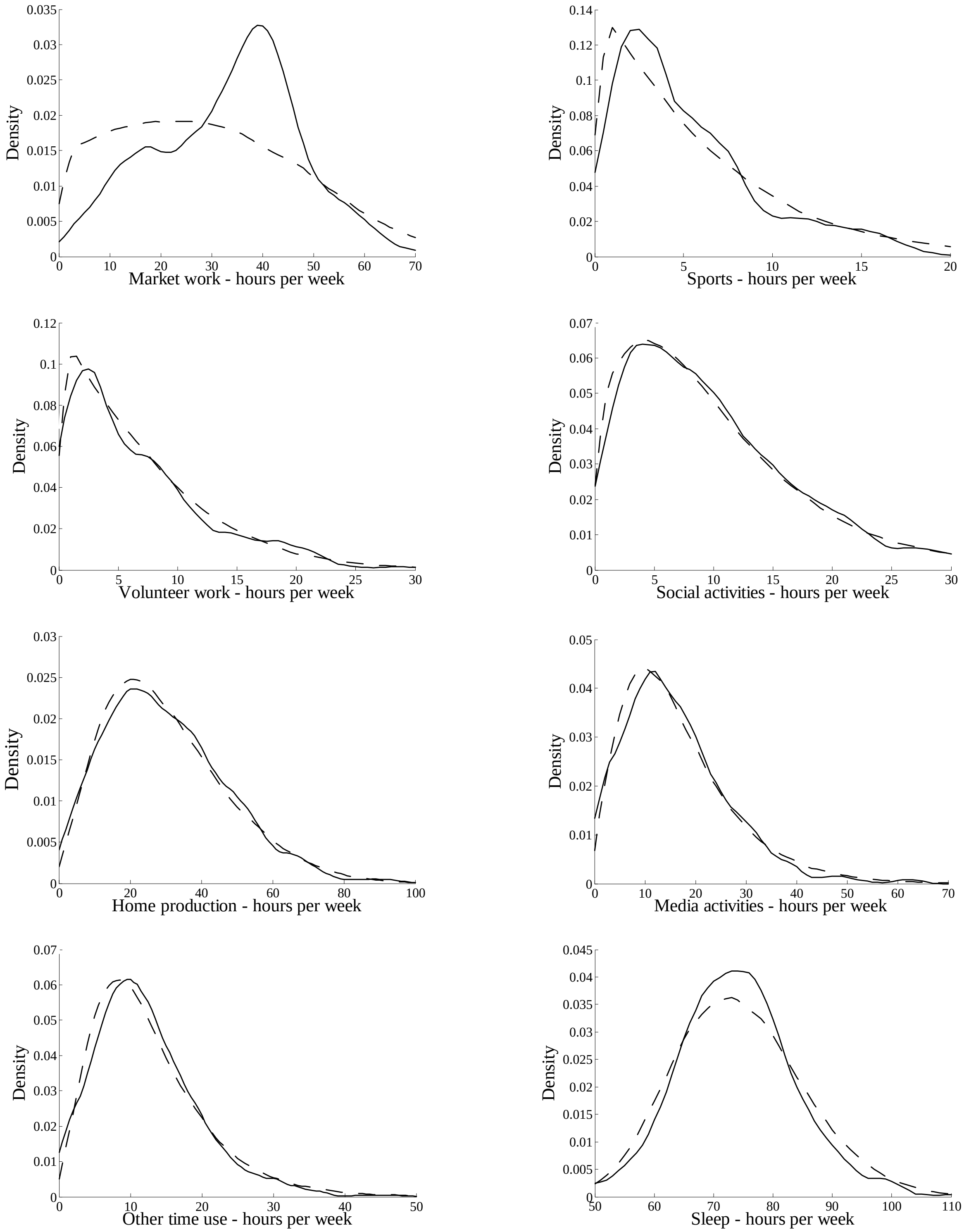

Figure 1: Females: Distributions of observed and predicted times. The solid lines are observed times and the dashed lines are predicted times. The diagrams for market work, sports, volunteer work and social activities refer only to individuals who participate in the activities.

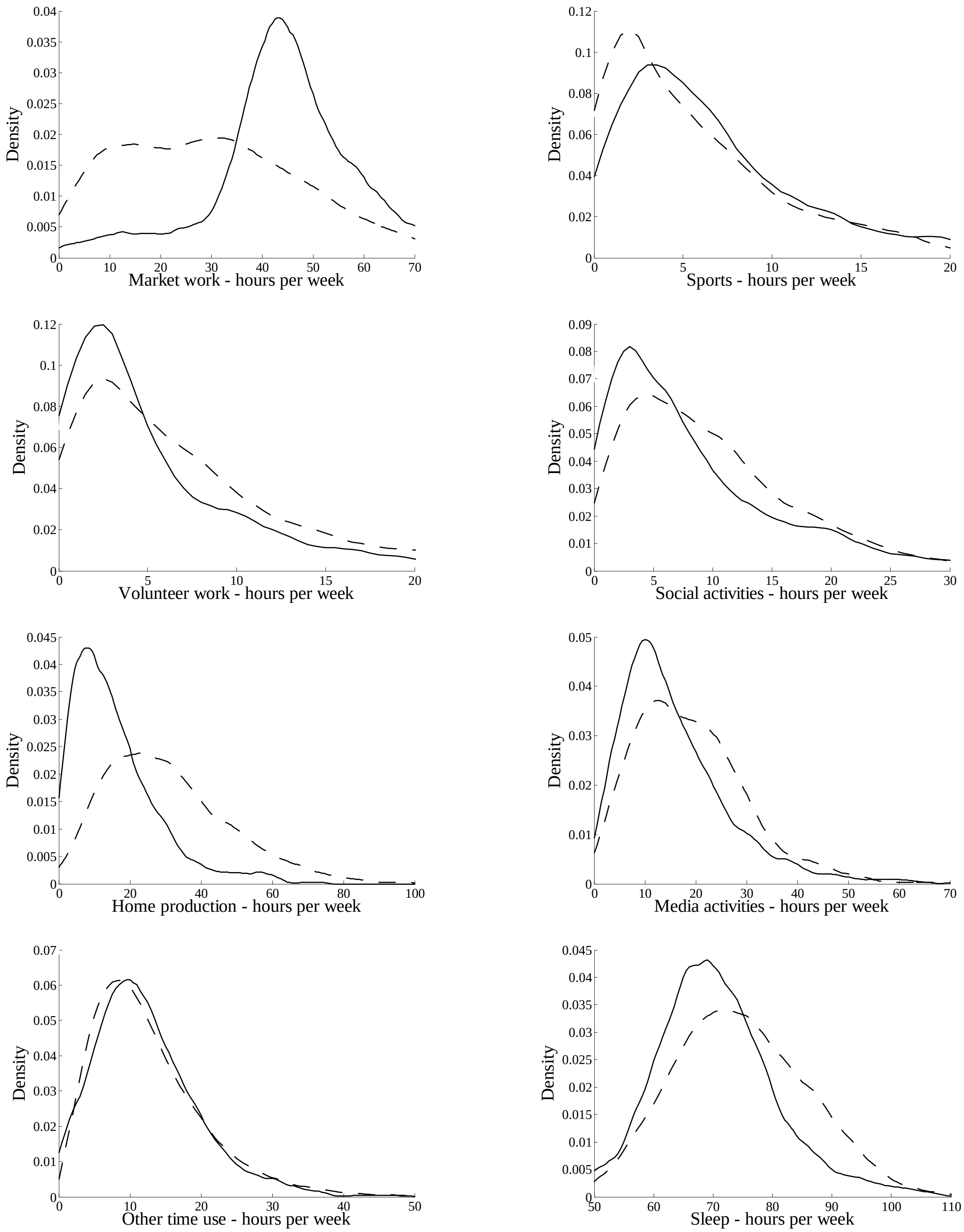

Figure 2: Males: Distributions of observed and predicted times. The solid lines are observed times and the dashed lines are predicted times. The diagrams for market work, sports, volunteer work and social activities refer only to individuals who participate in the activities.

Activity	Females					Males				
	Part.	T		ε		Part.	T		ε	
		Positive	All	Positive	All		Positive	All	Positive	All
Market work	0.007	0.35	0.53	-0.061	-0.0048	0.001	0.61	0.63	-0.066	-0.061
Sports	0	-0.013	-0.0052	-0.0194	0	-0.002	-0.098	-0.063	-0.0072	0
Volunteer Work	0.001	0.11	0.026	0.099	0	0.001	0.011	0.0076	0.037	0
Social activities	-0.003	-0.20	-0.22	-0.015	-0.017	-0.003	-0.14	-0.14	-0.010	-0.010
Home production	0	0.12		-0.0011		0	0.15		0.0056	
Media activities	0	0.0071		-0.0032		0	-0.054		-0.0014	
Other time use	0	-0.12		-0.0069		0	-0.10		-0.0022	
Sleep	0	-0.34		0.00032		0	-0.43		-0.0009	

Table 10: Margianal effect of age on participation, mean predicted times and median predicted wage elasticities.

Activity	Females					Males				
	Part.	T		ε		Part.	T		ε	
		Positive	All	Positive	All		Positive	All	Positive	All
Market work	0.017	1.48	1.84	-0.42	-0.063	-0.016	-1.33	-1.95	0.14	0.13
Sports	0.13	1.33	1.48	0.58	0	0.14	2.55	2.64	0.41	-1.48
Volunteer Work	-0.013	-0.72	-0.21	-1.46	0	-0.004	-0.35	-0.067	-0.76	0
Social activities	0.003	0.14	0.16	0.014	-0.0068	-0.003	-0.056	-0.075	0.044	0.055
Home production	0	-0.39		0.010		0	0.25		0.016	
Media activities	0	-3.35		-0.063		0	-2.20		-0.048	
Other time use	0	0.47		0.014		0	2.44		0.068	
Sleep	0	-0.0059		0.013		0	-1.02		0.012	

Table 11: Margianal effect of education on participation, mean predicted times and median predicted wage elasticities.

Activity	Females					Males				
	Part.	T		ε		Part.	T		ε	
		Positive	All	Positive	All		Positive	All	Positive	All
Market work	-0.089	-4.53	-6.46	0.96	0.31	-0.016	-1.12	-1.76	0.097	0.11
Sports	-0.023	0.13	-0.079	-0.34	0	0.025	1.12	0.77	-0.025	0
Volunteer Work	0.031	0.24	0.25	-0.20	0	0.026	0.49	0.23	2.42	0
Social activities	-0.004	0.63	0.52	0.054	0.14	0.009	0.55	0.56	0.056	0.050
Home production	0	5.38		0.17		0	0.97		0.033	
Media activities	0	0.043		0.065		0	0.080		0.015	
Other time use	0	0.12		0.032		0	0.17		0.023	
Sleep	0	0.23		0.0011		0	-1.02		-0.00055	

Table 12: Margianal effect of children on participation, mean predicted times and median predicted wage elasticities.