Deus ex machina wanted: time inconsistency of time consistency solutions in monetary policy.

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ABSTRACT. This paper argues that delegation (optimal institutional design) is not a solution to the dynamic inconcistency problem, and can even reinforce it. We show that 'optimal' delegation is not consistent with government's incentives. We solve for delegation schemes that are consistent with these incentives and find that they imply 'no delegation'. Introducing a cost of reappointing the central banker just postpones the problem, and can only solve it if the government is infinitely averse to changing central bank's contract. Our results hint to: (i) alternative explanations for good anti-inflationary performance; (ii) strengthening central bank independence and (iii) giving a more prominent role to Central Bank reputation building in fighting inflation.

1. Introduction

¹The optimal design of domestic monetary institutions has been the focus of an impressive amount of literature over the past quarter of a century, building on results on rules and discretion in monetary policy. The focus on rules and institutions analysed here goes well beyond its original dynamic inconsistency motivation. The basic insights are very simple: an authority conducting monetary policy when the socially optimal rate of output growth is higher than the natural one (due to some real distortion) is subject to a time consistency problem. A policy whereby it commits with respect to a rational private sector to a state-contingent policy rule is dynamically inconsistent: the policymaker has incentives to 'cheat' after expectations are formed in order to stimulate output by 'surprise inflation'. The time-consistent equilibrium that will arise will be one where policy is chosen discretionarily, and is suboptimal from the society's point of view. This trade-off has been identified by Kydland and Prescott (1977) and Barro and Gordon (1983) and

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¹Deus ex machina (Latin, 'god from the machinery'): Device in Greek theatre in which problems were resolved at the end of a play by the intervention of a god who was apparently brought down from Olympus. In fact he was moved by 'machinery' (a crane). It now refers to any contrived interposition in a novel, play, or film, and in general to any external, unexpected, last-minute resolution of a difficulty. Oxford Paperback Encyclopedia, © Oxford University Press 1998

is known as the 'inflation bias' of monetary policy, or as the 'dynamic inconsistency problem'. A first delegation-based solution to this problem² is found in a much celebrated earlier paper by Rogoff (1985), where it was argued that governments delegate monetary policy to a 'conservative', i.e. more inflation-averse than society, central banker. While this improved upon the discretionary equilibrium by reducing the inflation bias, it also introduced suboptimal stabilisation of shocks, introducing a tradeoff.

An ingenious way to get around this problem is found by Walsh (1995), Persson and Tabellini (1993) in a static framework and extended by i.a. Lockwood (1997), Lockwood Miller and Zhang (1997) and Svensson (1997) to a dynamic context. It consists of delegating monetary policy to an independent Central Bank, delegation being done by means of a 'contract' provided by the national government to its central bank. This contract induces an incentive scheme to the central bank (as e.g. linear penalties for excessive inflation) that would make it implement the optimal policy as the unique dynamically consistent equilibrium (the central bank acts discretionarily). Svensson (1997) shows how these 'contracts' can actually be thought of in terms of real-world inflation targeting regimes, such as those of New Zealand, Canada, Sweden, UK and others, whereby the central bank is assigned a lower inflation target than society's.

The argument of this paper is that while this literature is deemed to solve the dynamic inconsistency problem of monetary policy, it is subject to a dynamic inconsistency problem itself. While the literature recognizes the government has an incentive to 'cheat' when it conducts monetary policy, this incentive disappears when it comes to delegating. Somehow, in delegating the government is supposed to be able to implement an optimal policy, albeit non-compatible with its incentives. It is the purpose of this paper to find the delegation parameters chosen by the government based on its rationality, and prove these are different from the 'optimal' delegation parameters in an intuitive way. It turns out that the contract that the government would choose in our setup leads to implementation of the discretionary equilibrium, which, is entirely consistent with government's initial incentives. While optimal delegation is indeed desirable, nothing insures its implementability. Whether the government cheats or not is the same whether we talk about choosing monetary policy directly or designing an incentive scheme for a central bank. Making it costly for the government to revise the institutional arrangement does not solve the problem but merely postpones it.

Our results have immediate implications. Empirically, they question the alleged causality between 'inflation targeting' regimes and the success in fighting inflation. Theoretically, they hint to the need of a closer look into the design of incentives of the central bank, for one good reason: while research on the form of the optimal monetary policy rules has done much progress and is interesting on its own, finding the optimal rule is immaterial from a policy viewpoint as long as its implementation is highly improbable. Credibility problems of the type studied here or different arise in more ellaborate models of monetary policy, such as Clarida et al (1999) or Woodford (2000) and appropriate institutional design schemes to get around them are considered e.g. in Svensson and Woodford (2000) or Woodford (2000). Our

²This is not entirely correct as, perhaps interestingly, the idea of institutional design is present already in Footnote 19 of Barro and Gordon (1983).

results would equally apply in other frameworks as long as there are gains from commitment and a commitment technology is unavailable.

In the remainder we proceed as follows: Section 2 solves for the commitment and discretion equilibrium in a dynamic version of the Barro-Gordon model; Section 3 solves for optimal delegation parameters when delegation is done by an inflation contract and argues that optimal delegation is as non-implementable as optimal policy with commitment; Section 4 solves for 'perfect contracts' (in the game theoretical sense) by modelling the delegation stage explicitly and Section 5 concludes.

2. Commitment and discretion in a dynamic model

The model we use is a dynamic version of the Barro-Gordon (1983) model used by Svensson (1997). The model incorporates autoregressive dynamics ('persistence') in output of the sort introduced by Lockwood and Philippopoulos (1994). Persistence in output or unemployment is a well-known stylised fact (see Nelson and Plosser, 1982; Blanchard and Summers, 1986) and theoretical models based on unemployment hysteresis can be built in which this result is explained (a review of this research can be found in Lockwood and Philipopoulos). For our problem, this has strong implications (see Svensson 1997, or Lockwood 1997): the optimal delegation parameters we were mentioning in the introduction, whether inflation contracts or targets, become state-contingent. These equilibria are studied in detail in Svensson (1997) and Lockwood et al (1995), and we just reproduce the main results in our context for future use. We illustrate the argument by supposing, as in Svensson (1997), that the expectations-augmented aggregate supply curve is:

$$(2.1) y_t = \rho y_{t-1} + \alpha \left(\pi_t - \pi_t^e \right) + \varepsilon_t$$

In the above, y_t is the log of output, π_t is the inflation rate, π_t^e the inflation expected by the private sector and ε_t an iid supply shock with mean zero and variance σ^2 . The natural rate has been normalised to zero for convenience, hence y_t can be regarded as deviations from the natural rate. ρ is a constant parameter in the [0,1) interval capturing autoregressive dynamics in output. Suppose the private sector forms inflation expectations according to the rational expectations rule:

(2.2)
$$\pi_t^e = E[\pi_t \mid F_{t-1}] \equiv E_{t-1}\pi_t$$

The $E[. | \mathcal{F}_{t-1}]$ is the conditional expectation taken with respect to the information set \mathcal{F}_{t-1} , containing all the information available at time t-1, i.e. $\mathcal{F}_{t-1} = \{y_i, \pi_i, \varepsilon_i, \rho, \alpha\}_{i=1}^{t-1}$. Equation (1) will act as a constraint on the state variable in the future period, of the form: $y_t = \Gamma^d(y_{t-1}, \varepsilon_t)$. In the commitment case (to be discussed below), both equations (1) and (2) act as constraints, i.e. $y_t = \Gamma^c(y_{t-1}, \varepsilon_t)$.

The government's preferences are identical to those of society's and are assumed to concern inflation and output deviations from some optimal levels. Following the literature, these are supposed to be given by the following period loss function, where (π^*, y^*) is the socially optimal equilibrium and λ the weight on output stabilisation:

(2.3)
$$L_{t} = \frac{1}{2} \left[\left(\pi_{t} - \pi^{*} \right)^{2} + \lambda \left(y_{t} - y^{*} \right)^{2} \right]$$

Note that the assumption that $y^* > 0$ (which is the natural rate) gives rise to the inflation bias described before. Suppose further for simplicity that the government can perfectly control the inflation rate and that the timing of events at each t is as

follows: (0) either government commits to an optimal rule or delegates policy to an independent central bank, depending on the cases considered below; (i) expectations π_t^e are formed by (2); (ii) shocks ε_t are realised; (iii) π_t is chosen, if commitment has not taken place previously; (iv) y_t is fully determined.

When at (0) the government commits to a **state-contingent optimal rule** the policy is a solution to the problem:

(2.4)
$$\inf_{\{\pi_t\}, \{\pi_t^e\}} E_0 \left[\sum_{t=1}^{\infty} \beta^{t-1} L_t \right] \equiv v(y_0, \varepsilon_0)$$

$$s.t.(1), (2), (3)$$

Note that due to commitment the government can be regarded as choosing the inflation expectations since these are determined at (i) by government's decision at (0). Note that in this case uncertainty is not resolved when decision is being taken and the dynamic constraint correspondence (function) is $y_t = \Gamma^c(y_{t-1}, \varepsilon_t)$, comprising both (1) and (2). The Bellman equation associated to problem (4) is (where a superscript 'c' stands for 'commitment' throughout): (2.5)

$$v^{c}(y_{t-1}) = \inf_{\{\pi_{t}\}, \{\pi_{t}^{c}\}} E_{t-1} \left[\frac{1}{2} \left[(\pi_{t} - \pi^{*})^{2} + \lambda (y_{t} - y^{*})^{2} \right] + \beta v^{c}(y_{t}) \right], s.t.(1), (2)$$

Constraint (1) can be substituted directly in the loss function, and to constraint (2) we attach the Lagrangean multiplier θ_t to get the first order conditions for the right-hand side (do not assume a functional form for v^c (.) for the moment), with respect to π_t and π_t^e respectively:

$$\pi_{t} - \pi^{*} + \lambda \alpha (y_{t} - y^{*}) + \alpha \beta \frac{\partial v^{c}(y_{t})}{\partial y_{t}} - \theta_{t} = 0$$

$$-E_{t-1} \left[\lambda \alpha (y_{t} - y^{*}) + \alpha \beta \frac{\partial v^{c}(y_{t})}{\partial y_{t}} \right] + \theta_{t} = 0$$

Eliminating the Lagrange multiplier we obtain:

$$(2.6) \ \pi_t - \pi^* + \lambda \alpha \left(y_t - y^* \right) + \alpha \beta \frac{\partial v^c \left(y_t \right)}{\partial y_t} - E_{t-1} \left[\lambda \alpha \left(y_t - y^* \right) + \alpha \beta \frac{\partial v^c \left(y_t \right)}{\partial y_t} \right] = 0$$

Taking expectations of (6) at t-1 to pin down expected variables we obtain:

$$\pi_t^e = \pi^*$$

In order to solve for the Bellman equation we guess that the value function is quadratic (as the problem is linear-quadratic), of the form:

(2.7)
$$v^{c}(y_{t}) = \gamma_{0}^{c} + \gamma_{1}^{c}y_{t} + \gamma_{2}^{c}y_{t}^{2}, \text{ hence } \frac{\partial v^{c}(y_{t})}{\partial y_{t}} = \gamma_{1}^{c} + \gamma_{2}^{c}y_{t}$$

Substituting this, as well as the expressions for y_t and π_t^e in the first order condition we obtain the optimal state-contingent policy rule with commitment taking the value function as given:

(2.8)
$$\pi_t^c = \pi^* - \frac{\alpha \left(\lambda + \beta \gamma_2^c\right)}{1 + \alpha^2 \left(\lambda + \beta \gamma_2^c\right)} \varepsilon_t$$

Substitute these in (1) to get the state variable equation:

(2.9)
$$y_t^c = \rho y_{t-1} + \frac{1}{1 + \alpha^2 (\lambda + \beta \gamma_2^c)} \varepsilon_t$$

Now the functional Bellman equation can be solved by substituting in (5) the value function and π_t , y_t as obtained above and identifying coefficients on y_{t-1} , y_{t-1}^2 and the constant. However, as we are only interested in γ_1 and γ_2 we can use the Envelope Theorem on problem (5) taking into account that π_t^c , y_t^c is a minimum and treating y_{t-1} as a parameter. This would imply that:

$$\frac{\partial v\left(y_{t-1}\right)}{\partial y_{t-1}} = E_{t-1} \left[\frac{\partial L_t\left(\pi_t^c, y_t^c\right)}{\partial y_{t-1}} + \beta \frac{\partial v^c\left(y_t^c\right)}{\partial y_{t-1}} \right]
\Rightarrow \gamma_1^c + \gamma_2^c y_{t-1} = E_{t-1} \left[\rho \lambda \left((y_t - y^*) \right) + \beta \rho \left(\gamma_1^c + \gamma_2^c y_t \right) \right]
\Rightarrow \gamma_1^c + \gamma_2^c y_{t-1} = \rho^2 \lambda y_{t-1} - \rho \lambda y^* + \beta \rho \gamma_1^c + \beta \rho^2 \gamma_2^c y_{t-1}$$

Identifying coefficients and solving for γ_1, γ_2 we get:

$$\gamma_1^c = \frac{\rho \lambda}{\beta \rho - 1} y^*, \ \gamma_2^c = \frac{\rho^2 \lambda}{1 - \beta \rho^2}$$

and substituting in (8) we get the optimal policy rule under commitment:

(2.10)
$$\pi_t^c = \pi^* - \frac{\alpha \lambda}{1 + \lambda \alpha^2 - \beta \rho^2} \varepsilon_t$$

This is indeed a solution as the conditions of the stochastic verification principle (cf. Theorem 9.2 and Exercise 9.4 in Stokey and Lucas, 1989, Theorem 1 in Montrucchio, 2002) are satisfied in this simple case (shocks ε_t are iid with finite variance). This equilibrium is, however, not time consistent: the policymaker has incentives to deviate and stimulate output by inflating. The policy rule consistent with the incentives of the government can be obtained by solving for the **Markov Perfect Equilibrium** (see Fudenberg and Tirole 1991) or **discretionary equilibrium**. In this situation, at stage (0) nothing happens, and the government minimises the loss at (iii), after shocks are realised and expectations are formed, taking expectations as given:

(2.11)
$$\inf_{\{\pi_t\}} E_0 \left[\sum_{t=1}^{\infty} \beta^{t-1} L_t \right] \equiv v(y_0, \varepsilon_0)$$

$$s.t.(1), (3), \pi_t^e \text{ given}$$

the Bellman equation associated with this problem is (for 'd' superscript denoting 'discretion'):

$$(2.12) v^{d}(y_{t-1}) = E_{t-1} \inf_{\{\pi_{t}\}} \left[\frac{1}{2} \left[(\pi_{t} - \pi^{*})^{2} + \lambda (y_{t} - y^{*})^{2} \right] + \beta v^{d}(y_{t}) \right], s.t.(1)$$

where $\inf_{\{\pi_t\}}$ has been moved inside the expectations operator because when minimisation is done the supply shock realisation ε_t is known. The first order condition with respect to π_t , assuming a quadratic value function $v^d(y_t) = \gamma_0^d + \gamma_1^d y_t + \gamma_2^d y_t^2$ results in:

(2.13)
$$\pi_t - \pi^* + \alpha \left(\beta \gamma_1^d - \lambda y^*\right) + \alpha \left(\beta \gamma_2^d + \lambda\right) y_t = 0$$

and taking expectations at t-1 we obtain expected inflation as a function of the value function parameters as:

(2.14)
$$\pi_t^e = \pi^* - \alpha \left(\beta \gamma_1^d - \lambda y^*\right) - \alpha \rho \left(\beta \gamma_2^d + \lambda\right) y_{t-1}$$

This is a general result in the literature the inflation bias of the discretionary equilibrium features both an average term and a term dependent on past output realisations (third term above). Substituting this back in the first order condition using (1), we get the discretionary policy rule and hence output for a given value function:

$$(2.15)\pi_t^d = \pi^* - \alpha \left(\beta \gamma_1^d - \lambda y^*\right) - \frac{\alpha \left(\lambda + \beta \gamma_2^d\right)}{1 + \alpha^2 \left(\lambda + \beta \gamma_2^d\right)} \varepsilon_t - \alpha \rho \left(\beta \gamma_2^d + \lambda\right) y_{t-1}$$

$$y_t^d = \rho y_{t-1} + \frac{1}{1 + \alpha^2 \left(\lambda + \beta \gamma_2^d\right)} \varepsilon_t$$

Taking into account this is a minimum, y_{t-1} is a parameter when minimisation is done and now it affects π_t^d and π_t^e , we apply the Envelope Theorem to (12) to get:

$$\frac{\partial v^{d}(y_{t-1})}{\partial y_{t-1}} = E_{t-1} \left[\frac{\partial L_{t}(\pi_{t}^{d}, y_{t}^{d})}{\partial y_{t-1}} + \beta \frac{\partial v^{d}(y_{t}^{d})}{\partial y_{t-1}} \right]$$

$$\Rightarrow \gamma_{1}^{d} + \gamma_{2}^{d} y_{t-1} = E_{t-1} \left[-\alpha \rho \left(\beta \gamma_{2}^{d} + \lambda \right) \left(\pi_{t} - \pi^{*} \right) + \rho \lambda \left(y_{t} - y^{*} \right) + \beta \rho \left(\gamma_{1}^{d} + \gamma_{2}^{d} y_{t} \right) \right]$$

$$= \alpha^{2} \rho \left(\beta \gamma_{2}^{d} + \lambda \right) \left(\beta \gamma_{1}^{d} - \lambda y^{*} \right) - \rho \lambda y^{*} + \beta \rho \gamma_{1}^{d} + \left[\alpha^{2} \left(\beta \gamma_{2}^{d} + \lambda \right) + 1 \right] \rho^{2} \left(\beta \gamma_{2}^{d} + \lambda \right) y_{t-1}$$

where we have used again (1) and the obtained solutions for π_t^d, y_t^d . Identifying coefficient on y_{t-1} we get a second-degree equation in γ_2^d :

$$\alpha^{2}\rho^{2}\beta^{2} (\gamma_{2}^{d})^{2} + (2\alpha^{2}\rho^{2}\beta\lambda + \rho^{2}\beta - 1)\gamma_{2}^{d} + \rho^{2}\lambda (1 + \alpha^{2}\lambda) = 0$$

with solutions:

$$\gamma_{2\pm}^{d} = \frac{1 - 2\alpha^{2}\rho^{2}\beta\lambda - \rho^{2}\beta \pm \sqrt{\left(\rho^{2}\beta - 1\right)^{2} - 4\alpha^{2}\rho^{2}\beta\lambda}}{2\alpha^{2}\rho^{2}\beta^{2}}$$

which are real if the existence condition

$$\left(\rho^2\beta - 1\right)^2 - 4\alpha^2\rho^2\beta\lambda \ge 0$$

is satisfied. The relevant solution is $\gamma_{2_{-}}^{d} = \frac{1-2\alpha^{2}\rho^{2}\beta\lambda-\rho^{2}\beta-\sqrt{(\rho^{2}\beta-1)^{2}-4\alpha^{2}\rho^{2}\beta\lambda}}{2\alpha^{2}\rho^{2}\beta^{2}}$, denoted from now on as γ_{2}^{d3} . Given a solution for γ_{2}^{d} , a solution for γ_{1}^{d} is obtained by identifying the constant term giving

$$\gamma_{1}^{d} = -\frac{\alpha^{2} \rho \left(\beta \gamma_{2}^{d} + \lambda\right) \lambda y^{*}}{1 - \alpha^{2} \rho \beta \left(\beta \gamma_{2}^{d} + \lambda\right)}$$

Substituting back these values we get the discretionary policy rule and output as in (15) as functions of the initial parameters. Note that since γ_2^d is different from γ_2^c , the inflation solution features also a stabilisation bias (shock stabilisation is sub-optimal, by loooking at coefficients on ε_t).

3. Optimal contracts and their implementability

We now reproduce the result in Svensson (1997) and Lockwood et al (1995) concerning optimal delegation, although in a slightly more general framework. Namely, suppose at stage (0) the government delegates monetary policy to an independent central bank and does so by means of a linear contract in inflation

$$C_t = c_t \left(\pi_t - \pi^* \right)$$

³As this satisfies the verification principle condition $\lim_{t\to\infty} \beta^t E_0\left[v^d\left(y_t\right)\right]$, it is sufficient: this describes the unique value function. The other solution would necessarily violate the above condition.

Then, at stage (iii), the central bank will face the loss function plus the additional linear term in inflation above and will minimise this new loss function discretionarily. The question this literature asks is how to implement the commitment equilibrium (y^c, π^c) with discretionary policymaking by optimally designing the c_t , i.e. the marginal penalties (rewards) for additional inflation⁴. It turns out this is indeed possible for some value of c_t . Note that c_t will not be constant (a constant contract is suboptimal in the dynamic setup) but will be a function of shocks and past output $c_t(y_{t-1}, \varepsilon_t)$. Now suppose delegation has taken place at stage (0) and the central bank minimises the new loss function $L_t(.) + C_t$ taking delegation as given. We solve for the equilibrium as a function of contracts and see for what value of the latter is the resulting equilibrium identical to the commitment one (the approach usually taken in the literature to solve for optimal delegation parameters).

As this is still a Markov Perfect Equilibrium the Bellman equation of the Central Bank will be (where 'b' superscript stands for 'bank'):

$$v^{b}(y_{t-1}) = E_{t-1} \inf_{\{\pi_{t}\}} \left[\frac{1}{2} \left[(\pi_{t} - \pi^{*})^{2} + \lambda (y_{t} - y^{*})^{2} \right] + c_{t} (\pi_{t} - \pi^{*}) + \beta v^{b}(y_{t}) \right], s.t.(1)$$

Assuming again $v^b(y_t) = \gamma_0^b + \gamma_1^b y_t + \gamma_2^b y_t^2$ and taking the first order condition we get:

(3.1)
$$\pi_t - \pi^* + \alpha \left(\beta \gamma_1^b - \lambda y^*\right) + \alpha \left(\beta \gamma_2^b + \lambda\right) y_t + c_t = 0$$

and taking again expectations at t-1 we get the expected inflation in this regime as a function of the expected contract $c_t^e = E_{t-1}c_t$ under this regime (note that the expected contract appears as we allow c_t to be made contingent on ε_t).

(3.2)
$$\pi_t^e = \pi^* - \alpha \left(\beta \gamma_1^b - \lambda y^*\right) - \alpha \rho \left(\beta \gamma_2^b + \lambda\right) y_{t-1} - c_t^e$$

Substituting back we get the policy and output under this policy regime given the value function, using (1):

$$(3.3) \quad \pi_{t}^{b} = \pi^{*} - \alpha \left(\beta \gamma_{1}^{b} - \lambda y^{*}\right) - \frac{\alpha \left(\lambda + \beta \gamma_{2}^{b}\right)}{1 + \alpha^{2} \left(\lambda + \beta \gamma_{2}^{b}\right)} \varepsilon_{t} - \alpha \rho \left(\beta \gamma_{2}^{b} + \lambda\right) y_{t-1}$$

$$- \frac{\alpha^{2} \left(\lambda + \beta \gamma_{2}^{b}\right)}{1 + \alpha^{2} \left(\lambda + \beta \gamma_{2}^{b}\right)} c_{t}^{e} - \frac{1}{1 + \alpha^{2} \left(\lambda + \beta \gamma_{2}^{b}\right)} c_{t}$$

$$y_{t}^{b} = \rho y_{t-1} + \frac{1}{1 + \alpha^{2} \left(\lambda + \beta \gamma_{2}^{b}\right)} \varepsilon_{t} - \frac{\alpha}{1 + \alpha^{2} \left(\lambda + \beta \gamma_{2}^{b}\right)} \left(c_{t} - c_{t}^{e}\right)$$

To see what contracts implement the commitment equilibrium one needs to find the parameters of the value function, and we proceed as before by using the Envelope Theorem, the only additional complication being that y_{t-1} influences now also c_t and c_t^e , which has to be taken into account. Note that by definition and linearity of the model c_t and c_t^e differ only by a term in ε_t , so $\frac{\partial c_t}{\partial y_{t-1}} = \frac{\partial c_t^e}{\partial y_{t-1}}$ (hence y_{t-1} does not influence y_t^b through the contract and the influence on inflation is as below).

⁴Svensson (1997) also presents results fir the case with delegation to an inflation-targeting central bank, i.e. one where π^* in the loss function is replaced with a value π^b and the latter is object of optimal design. The results are largely the same, although different in this dynamic case, so we focus on contracts as the essence of the argument is the same.

Hence, as before, apply the Envelope Theorem to the Bellman equation to get:

$$\begin{split} \frac{\partial v^b \left(y_{t-1}\right)}{\partial y_{t-1}} &= E_{t-1} \left[\frac{\partial L_t \left(\pi_t^b, y_t^b\right)}{\partial y_{t-1}} + \frac{\partial \left[c_t \left(\pi_t^b - \pi^*\right)\right]}{\partial y_{t-1}} + \beta \frac{\partial v^b \left(y_t^b\right)}{\partial y_{t-1}} \right] \Rightarrow \\ \gamma_1^b + \gamma_2^b y_{t-1} \!\! \left(3.4 \!\! \right) & E_{t-1} \left\{ \begin{array}{c} \left[-\alpha \rho \left(\beta \gamma^b + \lambda\right) - \frac{\partial c_t^e}{\partial y_{t-1}} \right] \left(\pi_t - \pi^*\right) + \rho \lambda \left(y_t - y^*\right) + \\ + \frac{\partial c_t^e}{\partial y_{t-1}} \left(\pi_t - \pi^*\right) + c_t \left[-\alpha \rho \left(\beta \gamma^b + \lambda\right) - \frac{\partial c_t^e}{\partial y_{t-1}} \right] + \beta \rho \left(\gamma_1^b + \gamma_2^b y_t\right) \end{array} \right\} \end{split}$$

We can already look at the optimal contract. By definition, this should be such that $\pi_t^e = \pi^*$ as in the optimal rule. But this can only happen in this equilibrium if, from (17):

(3.5)
$$c_t^e = \alpha \left(\lambda y^* - \beta \gamma_1^b \right) - \alpha \rho \left(\beta \gamma_2^b + \lambda \right) y_{t-1}$$

Using this and $\pi_t^e = \pi^*$, (19) becomes after replacing and taking expectations:

$$\gamma_1^b + \gamma_2^b y_{t-1} = \rho^2 \lambda y_{t-1} - \rho \lambda y^* + \beta \rho \gamma_1^b + \beta \rho^2 \gamma_2^b y_{t-1}$$

which, gives the same value function as in the commitment case, i.e.:

$$\gamma_1^{bo} = \gamma_1^c = \frac{\rho\lambda}{\beta\rho - 1} y^*, \gamma_2^{bo} = \gamma_2^c = \frac{\rho^2\lambda}{1 - \beta\rho^2}$$

We need to look at the 'unexpected' part of the contract, but it is easily seen that for $\gamma_i^{bo} = \gamma_i^c$ the shock stabilisation coefficients in π_t^b above are the same as in the optimal rule π_t^c , hence the contract needs not be made contingent upon ε_t . Note that in contrast to the literature we did not assume this, but found it by optimality. The government delegates to the central bank such that the value function of the latter becomes identical to its own value function when committing. Hence the optimal contract, such that the commitment equilibrium is implemented in the discretionary case is:

(3.6)
$$c_t^o = \frac{\alpha \lambda}{1 - \beta \rho} y^* - \frac{\alpha \lambda}{1 - \beta \rho^2} \rho y_{t-1}, \forall t \ge 1$$

A natural question which we have formulated in the introduction arises now: why a government that has incentives to deviate from the optimal policy rule π_t^c would be able to implement c_t^o as given by (21)? The critical step is taken when passing from equation (19) to (20). There, it is obvious that what makes the government choose a contract leading to $\pi_t^c = \pi^*$ are not its incentives; these incentives make it deviate from the optimal policy in the first place (this was identified by Kydland and Prescott, 1977 and Barro and Gordon, 1983). What would make it act as such would be a kind of benevolence (or commitment mechanism) that would be hard to justify given that it did not have it before. As McCallum (1995, p 210) puts it, 'if a commitment technology does not exist, then it does not exist'.

4. Government incentives and cheating by delegation

We choose to model the choice of the contract based on individual rationality of the government. This would imply that we model explicitly what happen at stage (0), instead of merely assuming the government is a sort of 'Deus ex machina' that intervenes at the right time by selecting the right equilibrium. Hence, at stage (0) the government would choose the sequence $\{c_t\}$, its new control variable given the fact that instrument independence has been granted to the central bank. It would do so by backward induction, not only in the sense that the model has to be solved

by dynamic programming, but also because it observes the way the central bank chooses its policy instrument at stage (iii) in the Markov Perfect Equilibrium we described above. Then (17) and (18) become the government's dynamic constraints and it will solve:

(4.1)
$$\inf_{\{c_t\}} E_0 \left[\sum_{t=1}^{\infty} \beta^{t-1} L_t(\pi_t^b(c_t), y_t^b(c_t)) \right] \equiv v^g(y_0, \varepsilon_0)$$

$$s.t.(17), (18), c_t^e \text{ given}$$

The associated Bellman equation will be (note that minimisation is done before uncertainty is resolved):

(4.2)

$$v^{g}(y_{t-1}) = \inf_{\{c_{t}\}} E_{t-1} \left[\frac{1}{2} \left[\left(\pi_{t}^{b} - \pi^{*} \right)^{2} + \lambda \left(y_{t}^{b} - y^{*} \right)^{2} \right] + \beta v^{g}(y_{t}^{b}) \right], s.t.(17), (18)$$

The first order condition with respect to c_t is, treating the parameters of the value function in the central bank's problem γ_i^b as given and guessing that $v^g(y_t)$ is quadratic:

$$-\frac{1}{1+\alpha^{2}\left(\lambda+\beta\gamma_{2}^{b}\right)}\left(\pi_{t}^{b}-\pi^{*}\right)-\frac{\alpha\lambda}{1+\alpha^{2}\left(\lambda+\beta\gamma_{2}^{b}\right)}\left(y_{t}^{b}-y^{*}\right)+\beta\left(\gamma_{1}^{g}+\gamma_{2}^{g}y_{t}\right)\left[-\frac{\alpha\lambda}{1+\alpha^{2}\left(\lambda+\beta\gamma_{2}^{b}\right)}\right]=0$$

$$(4.3) \qquad \Rightarrow (\pi_t^b - \pi^*) + \alpha \lambda (y_t^b - y^*) + \alpha \beta (\gamma_1^g + \gamma_2^g y_t) = 0$$

Taking expectations at t-1 substituting for π_t^b, y_t^b we obtain:

$$-\alpha \left(\beta \gamma_1^b - \lambda y^*\right) - \alpha \rho \left(\beta \gamma_2^b + \lambda\right) y_{t-1} - c_t^e + \alpha \lambda \rho y_{t-1} - \alpha \lambda y^* + \alpha \beta \left(\gamma_1^g + \gamma_2^g \rho y_{t-1}\right) = 0$$

Hence

$$(4.4) c_t^e = \alpha\beta \left[\left(\gamma_1^g - \gamma_1^b \right) + \left(\gamma_2^g - \gamma_2^b \right) \rho y_{t-1} \right]$$

Substituting back in the first order condition we can get after some algebra:

$$(4.5) c_t^* = \alpha\beta \left[\left(\gamma_1^g - \gamma_1^b \right) + \left(\gamma_2^g - \gamma_2^b \right) \rho y_{t-1} + \frac{\left(\gamma_2^g - \gamma_2^b \right)}{1 + \alpha^2 \left(\lambda + \beta \gamma_2^g \right)} \varepsilon_t \right]$$

We need to find the parameters of the value function of the government γ_i^g . To do this apply the Envelope Theorem to (23) to get:

$$\gamma_{1}^{g} + \gamma_{2}^{g} y_{t-1} = E_{t-1} \left\{ \begin{bmatrix} -\alpha \rho \left(\beta \gamma^{b} + \lambda \right) - \alpha \beta \rho \left(\gamma_{2}^{g} - \gamma_{2}^{b} \right) \right] \left(\pi_{t}^{b} - \pi^{*} \right) + \\ +\rho \lambda \left(y_{t}^{b} - y^{*} \right) + \beta \rho \left(\gamma_{1}^{g} + \gamma_{2}^{g} y_{t}^{b} \right) \end{bmatrix} \right\}
(4.6) = E_{t-1} \left\{ -\alpha \rho \left(\beta \gamma^{g} + \lambda \right) \left(\pi_{t}^{b} - \pi^{*} \right) + \rho \lambda \left(y_{t}^{b} - y^{*} \right) + \beta \rho \left(\gamma_{1}^{g} + \gamma_{2}^{g} y_{t}^{b} \right) \right\}$$

But note this is exactly the same as the equation we got in the Markov Perfect equilibrium without delegation (i.e. for γ_1^d, γ_2^d)! The only difference is that it now has to hold for the (Markov Perfect) equilibrium values of inflation and output we found in central bank's problem. Intuitively, note that we know one value of (γ_1^g, γ_2^g) for which the above holds in this case: it is exactly (γ_1^d, γ_2^d) , the value function when there is no delegation, which would also imply that (π_t^b, y_t^b) are the equilibrium values without delegation, (π_t^d, y_t^d) . This would mean $\{c_t\}$ is identically zero for all t and this is indeed a solution to this problem. To see this, solve explicitly for the value function parameters by substituting in (27) the expressions

 π_t^b, y_t^b evaluated at the c_t^* found in (26) as a solution to government's optimisation, the latter expressions being:

$$\pi_{t}^{b}\left(c_{t}^{*}\right) = \pi^{*} - \alpha\left(\beta\gamma_{1}^{b} - \lambda y^{*}\right) - \frac{\alpha\left(\lambda + \beta\gamma_{2}^{b}\right)}{1 + \alpha^{2}\left(\lambda + \beta\gamma_{2}^{b}\right)} \varepsilon_{t} - \alpha\rho\left(\beta\gamma_{2}^{b} + \lambda\right) y_{t-1}$$

$$-\alpha\beta\left(\gamma_{1}^{g} - \gamma_{1}^{b}\right) - \alpha\beta\left(\gamma_{2}^{g} - \gamma_{2}^{b}\right) \rho y_{t-1} - \frac{\alpha\beta\left(\gamma_{2}^{g} - \gamma_{2}^{b}\right)}{\left[1 + \alpha^{2}\left(\lambda + \beta\gamma_{2}^{g}\right)\right]\left[1 + \alpha^{2}\left(\lambda + \beta\gamma_{2}^{b}\right)\right]} \varepsilon_{t}$$

$$(4.7)\pi^{*} - \alpha\left(\beta\gamma_{1}^{g} - \lambda y^{*}\right) - \frac{\alpha\left(\lambda + \beta\gamma_{2}^{g}\right)}{1 + \alpha^{2}\left(\lambda + \beta\gamma_{2}^{g}\right)} \varepsilon_{t} - \alpha\rho\left(\beta\gamma_{2}^{g} + \lambda\right) y_{t-1}$$

$$y_{t}^{b}\left(c_{t}^{*}\right) = \rho y_{t-1} + \frac{1}{1 + \alpha^{2}\left(\lambda + \beta\gamma_{2}^{g}\right)} \varepsilon_{t}$$

But these are the same as (15), the solution we had when the government discretionarily chose monetary policy without delegating it, the only change being again the parameters of the value function. What this means is that as long as the government optimises and chooses c_t^* as given by (26) the parameters of its value function fully determine (π_t^b, y_t^b) , the equilibrium in Central Bank's game too, and makes it equal to the equilibrium values under discretion found before in (15), (π_t^d, y_t^d) . This is intuitive - the same incentives that made the government choose policy discretionarily when its control variable was π_t make it choose c_t^* now such that the same equilibrium is implemented. By solving explicitly for the value function parameters now one would get the same expression as (γ_1^d, γ_2^d) , and this is unique! The last thing to note is obvious: by definition the equilibrium in central bank's game with delegation is identical to the one without delegation in case c_t^* is identically zero, i.e.:

(4.8)
$$\pi_t^b(c_t^*) = \pi_t^d, y_t^b(c_t^*) = y_t^d \Rightarrow c_t^* = 0, \forall t$$

This is entirely consistent with what happens in Central Bank's game: for $c_t^* = 0$, the parameters of its value function are simply the same as the ones of the government in the discretionary case without delegation, i.e.

$$\gamma_i^b = \gamma_i^d, \forall i \in \{1, 2\}$$

Another (equivalent but more computationally demanding) way to obtain with result would be analytical. If one solves explicitly for the value function parameters in the central bank's delegated game, γ_i^b from (19) and gets these as functions of the delegation parameters, i.e. parameters of the function c_t (these are taken as given when bank chooses policy). An important thing to note (results are available at request but this is trivial) is that $\gamma_i^b(c_t) = \gamma_i^d$ if and only if $c_t = 0$. On the other hand, by (27) or (28) the value function of the government when it delegates γ_i^g is the same as the value function when it has chosen monetary policy directly, γ_i^d . By postulating a functional form for c_t (e.g.. it is linear feed-back in output and future shocks as the problem is linear-quadratic) such as $c_t = c_c + c_y y_{t-1} + c_\varepsilon \varepsilon_t$ one can substitute this in (26), which becomes after all these changes:

$$c_{c}+c_{y}y_{t-1}+c_{\varepsilon}\varepsilon_{t}=\alpha\beta\left[\left(\gamma_{1}^{d}-\gamma_{1}^{b}\left(c_{c},c_{y},c_{\varepsilon}\right)\right)+\left(\gamma_{2}^{d}-\gamma_{2}^{b}\left(c_{c},c_{y},c_{\varepsilon}\right)\right)\rho y_{t-1}+\frac{\left(\gamma_{2}^{d}-\gamma_{2}^{b}\left(c_{c},c_{y},c_{\varepsilon}\right)\right)}{1+\alpha^{2}\left(\lambda+\beta\gamma_{2}^{d}\left(c_{c},c_{y},c_{\varepsilon}\right)\right)\varepsilon_{t}}\right]$$

By identifying coefficients, one sees immediately that $c_c = 0, c_y = 0, c_{\varepsilon} = 0$ is a solution, which is what we also got before, implying $c_t^* = 0$.

Now, this is obviously different from the optimal contracts we solved for before in (21), and in an intuitive way. Our result is just an instance of the government's incentives to cheat in the first place. These 'discretionary' or Markov Perfect contracts are different from the optimal contracts in the same way in which discretionary policy was different from optimal policy (with commitment). If the government has incentives to cheat, we show these are at work no matter if it chooses policy or the institution that chooses policy. The latter form is probably more subtle but leads to exactly the same outcome, preserving the assumption about the private sector forming rational expectations. The government has the same value function as when it chooses monetary policy directly, and now is able to influence also the value function of the central bank. Based on its individual rationality, of course it would choose that contract (in the form of a penalty for additional inflation) that makes the value function of the central bank be consistent with its preferred equilibrium. It turns out this penalty is zero.⁵

While sometimes the literature acknowledges the possibility of a failure of enforcement of optimal delegation (e.g. Walsh 1995 footnote 5), the counterargument, albeit implicit, is that constitutions are always binding. But that again is a solution by assumption, let aside that it might not be true⁶. Another way to interpret this argument is that changing institutions is costly, as Jensen (1997) does. Note that in our model there is no cost of changing the institution yet. We now introduce such a cost and explain briefly the intuition.

4.1. Reappointment costs. For example, suppose the government faces a

cost in changing its delegation parameters, c_t . We will model this in two ways: first, consider that the government dislikes deviations of the contract it chooses from the optimal contract c_t^o . This means that in a first period it was able to implement this contract, and now, when it has the opportunity to renege on it, it faces a cost. We model this as a quadratic cost, not necessarily in monetary terms, but easily interpretable as a loss of reputation, e.g. for financial markets. Then, at stage (0), the government would face the following loss function, again taking as constraints the choice of the central banker given delegation:

(4.9)
$$L_{t} = \frac{1}{2} \left[\left(\pi_{t}^{b} - \pi^{*} \right)^{2} + \lambda \left(y_{t}^{b} - y^{*} \right)^{2} \right] + \frac{\delta}{2} \left(c_{t} - c_{t}^{o} \right)^{2}$$

For simplicity, focus on the case without persistence outlined in the Appendix. In this case the optimal contract c_t^o is constant, $c^o = \lambda \alpha y^*$. Substituting this and the expressions for equilibrium in the Central Bank problem and minimising with respect to c_t (noting also that now the problem boils down to period-by-period minimisation as $\rho = 0$), the first order condition after some algebra results in the

⁵It is intuitive by the same argument that in the case of delegation to an inflation-targeting central bank, the solution for the inflation target with which the government will delegate will not be the optimal one, obtained by Svensson (1997). It will instead be just π^* , the target that leads to the discretionary equilibrium.

⁶Moreover, McCallum (1995) argues that institutions need not bind, giving the most prominent example: the gold standard in the US. This has not been abandoned de jure, although de facto this happenned a while ago. Moreover, New Zealand's 'contract' has been changed a few times as well. See the discussion in Jensen 1997.

following equilibrium contract:

(4.10)
$$c^* = \frac{\delta (1 + \alpha^2 \lambda)}{1 + \delta (1 + \alpha^2 \lambda)} c^o = \frac{\delta (1 + \alpha^2 \lambda)}{1 + \delta (1 + \alpha^2 \lambda)} \lambda \alpha y^*$$

This is always smaller than the optimal contract as the coefficient is less than one. The intuition is straightforward: as δ , the 'reappointment aversion' parameter, goes to zero, c^* is simply 0, the equilibrium contract without any reappointment costs. As the aversion to institutional change (or the difficulty to implement this) increases, the equilibrium contract approaches the optimal contract. It is trivial to see that $\lim_{\delta \to \infty} c^* = c^o$. 'Infinite' aversion to institutional change might sustain the optimal contract as subgame perfect.

Second way to look at this problem is to consider that the government 'smoothes' the contract, i.e. it dislkies deviations of this period's contract from last period's contract. Equivalently, it values institutional stability. In this case, the period loss function is:

(4.11)
$$L_{t} = \frac{1}{2} \left[\left(\pi_{t}^{b} - \pi^{*} \right)^{2} + \lambda \left(y_{t}^{b} - y^{*} \right)^{2} \right] + \frac{\varphi}{2} \left(c_{t} - c_{t-1} \right)^{2}$$

By same method as above, looking again at the no-persistence case, the solution will be a homogenous difference equation:

$$c_t^{**} = \frac{\varphi\left(1 + \alpha^2 \lambda\right)}{1 + \varphi\left(1 + \alpha^2 \lambda\right)} c_{t-1}^*$$

As the coefficient is less than one, the solution is stable and solving the equation gives:

(4.12)
$$c_t^{**} = \left[\frac{\varphi(1+\alpha^2\lambda)}{1+\varphi(1+\alpha^2\lambda)}\right]^t c_0$$

This converges assymptotically to zero, so the contract will in the limit be the Markov perfect one we solved for before, i.e. $c_t^{**} = 0$ (again, in the limit). Even if governments implements in the first period the optimal contract $c_0 = c^o$, it will follow its incentives and start decrease the penalty to its preffered zero level, the speed with which it does so being dependent on how much it values institutional stability, i.e. on φ .

The results for the persistence case would be different but the main intuition should remain. Reappointment costs do not solve the problem, they merely postpone it.

5. Conclusions

Theoretical work in the area of optimal design of monetary institutions seems to have reached a consensus. By appropriately delegating monetary policy to an instrument-independent central bank (whether via an inflation contract or a target) the government is able to achieve an equilibrium that it would not otherwise be able to achieve. By giving up the control of inflation (or more realistically, of a policy instrument that influences inflation), the government is not tempted to cheat by creating surprise inflation in order to stimulate output. Then the appropriately delegated central bank takes care of monetary policy and is able to achieve the 'commitment' equilibrium choosing policy discretionarily.

The key word in the argument above is 'appropriately'. As it is the government who chooses the institution, there is nothing to ensure that it would choose the 'optimal' one, unless it acts as a *Deus ex machina* does in an antique tragedy. But if it has this ability (or benevolence) it is hard to understand why it does not use it when choosing policy. Once we accept this, it is not surprising that it will choose that institution (which, according to case means 'inflation target' or 'marginal penalty for additional inflation') that implements its preferred equilibrium, and that is not Pareto optimal.

Reappointment costs (i.e. costs of changing the institutional arrangement for the central bank) do not solve the problem but rather postpone it, as we argue in a last section based on two ways to model such costs. To sustain the optimal contract over time, an infinite aversion to institutional change is needed, but that is hard to match with reality. One important thing to note is the way in which the critique we formulate is different from that of McCallum (1995). We do not merely argue that the optimal delegation will not be enforced; our result implies more, i.e. that optimal delegation will not be chosen in the first place.

Note that our result is not a critique of central bank independence, nor of monetray policy delegation à la Rogoff, per se. Institutional arrangements in which independence of the central bank is being given prominence are useful in order to avoid political pressure on the central bank. But designing such an arrangement 'optimally' is far from being a panaceum in order to avoid such political problems and is potentially a way to conserve the possibility of such political pressures. Another implication of our analysis is in terms of inflation performance in inflation-targeting countries. One could argue that the low inflation we observe over the past years is an effect of such institutional arrangements and of course that is still subject to debate as it is a very strong causal statement, which needs careful empirical investigation which we leave for future research. Incidentally, observe that countries in which Central Banks are not 'inflation targeters' are still successful in fighting inflation (the Fed is the most obvious example). Alternative explanations for success in fighting inflation can easily be found: a cheap one would be a sequence of favourable shocks making the temptation to inflate superfluous.

Relatedly, if one argues inflation targeting regimes can be interpreted as an inflation target à la Svensson, i.e. equal to society's preferred inflation minus the average inflation bias, an immediate question arises. How can one be sure that the inflation target specified in the status of a Central Bank (2%, to give an example) is lower than society's, and it is so by the amount that makes the equilibrium optimal? Isn't it equally plausible that this target is dully equal to the socially optimal one? In the latter case the inflation bias problem would not be solved, and if this is a description of reality the fact that we are passing through a period of low inflation might be due to other factors. A sensible explanation has been proposed by Blinder (1997). He argues that central bankers 'learned' and they do not attempt to increase output above the natural rate by surprise inflation. In the model here, the commitment solution can be obtained by having the long-run average of output as a target, i.e. $E[y_t] = \rho y_{t-1}$, or in the static version below by having a target equal to the natural rate in the loss function of the bank. To explain this, one would need a model of learning by the central bank, but note that full instrument independence of the central bank is still crucial. Our results come just to strengthen arguments for this independence. These explanations for

the apparent success in fighting inflation, together with reputation-building by the central banks, seem more plausible than optimal delegation by a government that is benevolent when delegating, but could not be so were it be able to control monetary policy directly. These questions open the field for interesting empirical problems, investigating e.g. the causality between the institutional arrangements we observe and the success of monetary policy in fighting inflation.

Appendix A. The case without output persistence

For the sake of transparency and simplicity, as well as ease of comparison with some of the related literature (e.g. Walsh 1995, Svensson 1997, Persson and Tabellini 1993), we consider the case without output persistence. In this case $\rho=0$ and equation (1) becomes an usual Lucas supply curve. As there is no intertemporal constraint the dynamic problem (4) boils down to minimising period-by period the loss function in (3), hence the value function becomes a constant and the coefficients γ_1 and γ_2 become identically zero in all regimes. Replacing ρ, γ_1, γ_2 with zero in (8) and (9) we get the optimal policy under commitment as:

(A.1)
$$\pi_t^c = \pi^* - \frac{\alpha \lambda}{1 + \alpha^2 \lambda} \varepsilon_t$$
$$y_t^c = \frac{1}{1 + \alpha^2 \lambda} \varepsilon_t$$
$$\pi_t^e = \pi^*$$

By the same token, from (15) the discretionary equilibrium is now:

(A.2)
$$\pi_t^d = \pi^* - \alpha \lambda y^* - \frac{\alpha \lambda}{1 + \alpha^2 \lambda} \varepsilon_t$$

$$y_t^d = \frac{1}{1 + \alpha^2 \lambda} \varepsilon_t$$

$$\pi_t^e = \pi^* + \alpha \lambda y^*$$

Optimal contracts would now be constant to eliminate the average inflation bias $\alpha \lambda y^*$:

$$(A.3) c_{+}^{o} = c^{o} = \alpha \lambda y^{*}$$

Similarly an optimal inflation target 'a la Svensson' doing the same job would be $\pi^o = \pi^* - \alpha \lambda y^*$.

Given delegation took place at stage zero with a linear contract $c(\pi_t - \pi^*)$, where the contract can be made contingent on shocks, the equilibrium in the central bank's problem would be from (18):

(A.4)
$$\pi_t^b = \pi^* - \alpha \lambda y^* - \frac{\alpha \lambda}{1 + \alpha^2 \lambda} \varepsilon_t - \frac{\alpha^2 \lambda}{1 + \alpha^2 \lambda} c^e - \frac{1}{1 + \alpha^2 \lambda} c$$
$$y_t^b = \frac{1}{1 + \alpha^2 \lambda} \varepsilon_t - \frac{\alpha}{1 + \alpha^2 \lambda} (c - c^e)$$

Substituting this back in the loss function of the government (3) and minimising with respect to c, to correspond to the choice of the policy regime by the government, gives the first order condition:

$$0 = -\frac{1}{1+\alpha^2\lambda} \left[\alpha\lambda y^* - \frac{\alpha\lambda}{1+\alpha^2\lambda} \varepsilon_t - \frac{\alpha^2\lambda}{1+\alpha^2\lambda} c^e - \frac{1}{1+\alpha^2\lambda} c \right] - \frac{\alpha\lambda}{1+\alpha^2\lambda} \left[\frac{1}{1+\alpha^2\lambda} \varepsilon_t - \frac{\alpha}{1+\alpha^2\lambda} (c-c^e) - y^* \right]$$

Taking expectaitons at t-1 of this gives $c^e=0$ and substituting back gives c=0 as government's choice. Similarly, one can show that if delegation is done by an inflation target, i.e. asigning to the central bank a loss function of the form:

$$L_{t} = \frac{1}{2} \left[\left(\pi_{t} - \pi^{T} \right)^{2} + \lambda \left(y_{t} - y^{*} \right)^{2} \right]$$

the inflation target chosen by the government will by the same method as above be $\pi^T = \pi^*$, the socially optimal inflation, as opposed to $\pi^o = \pi^* - \alpha \lambda y^*$, the desirable 'optimal inflation target' found by Svensson (1997). Hence, the average inflation bias will not be eliminated by delegation even in this simple static model. The main policy insights are the same as in the more general dynamic model discussed in text.

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