Incomplete Fiscal Rules with Imperfect Enforcement[1]

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Abstract

This paper analyses the effect of limits on fiscal deficits when fiscal policy outcomes depend on automatic stabilizers and when fiscal rules lack perfect credibility. The model developed, which includes interactions between monetary and fiscal policy, provides theoretical support for existing arguments that fiscal rules contracted on a structural deficit will be welfare-enhancing relative to rules written on the actual deficit. The latter rules would result in a procyclical bias in fiscal policy, as well as a contractionary bias in monetary policy. Contrary to existing arguments, the model also suggests that rules written on the structural deficit may ultimately be more credible than those written on the actual deficit. The reason for this is that rules written on the actual fiscal deficit risk running into a *credibility trap*; higher marginal penalties will be necessary when initial credibility of enforcement is imperfect, but announcing a higher penalty for violating a fiscal rule can actually reduce credibility if the penalty is disproportionately large relative to the violation.

1 Introduction

Researchers in recent years have devoted a great deal of attention to the idea of establishing rules for fiscal policy in the form of numerical limits on the accumulation of debt and deficits. Rules have been proposed both to ensure that governments do not accumulate excessive amounts of debt, and, in contexts such as the European Monetary Union, to address the problem that individual national fiscal authorities may fail to internalize the effect of their policies on other member states and on a common central bank. One common criticism of fiscal rules is that because they tend to be contracted on the actual fiscal deficit, rather than the cyclically adjusted deficit, they may induce a procyclical bias in fiscal policy. Fiscal rules will provide insufficient discipline for governments when economic conditions are favorable while they will force governments to take an excessively contractionary fiscal policy stance during recessions. This has long been a criticism of the Stability and Growth Pact for EMU states, as it has been feared that a firm application of the pact would interfere with the operation of automatic stabilizers for fiscal policy. In response to this problem a number of authors have proposed modifying the Stability Pact rules to focus on the structural (cyclically adjusted) deficit, and such proposals have become more frequent since 2001 in a period of slow economic growth.[1] However, it has also been argued that fiscal rules written on the actual deficit may be more credible, which provides a potential argument for not modifying the Stability and Growth Pact.

In this paper we develop a model for analyzing the effect of numerical limits on fiscal deficits when fiscal policy outcomes depend upon automatic stabilizers and when fiscal rules lack perfect credibility. Our framework includes a monetary authority, a fiscal authority, a principal who can write contracts with each of these actors, and interactions between monetary and fiscal policy. This framework can be used to analyze monetary-fiscal interactions within a single country. It can also be used to draw insights for settings like a monetary union where there is a national fiscal authority facing a supranational monetary authority, as long as output and inflation outcomes depend upon choices made by both authorities. We consider two alternative fiscal rules[2]: a linear deficit contract imposed on the structural deficit versus a

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[1]For a review see Artis (2002). The European Commission proposed several mechanisms "to improve the interpretation of the Stability and Growth Pact" in October 2002.

[2]A note on terminology is in order. In the monetary policy literature, by 'rule' one means a law of motion

linear deficit contract that is imposed on the 'actual' deficit and which also includes an escape clause for severe economic shocks.[3] The former rule is an example of a complete contract, while the actual deficit contract is an incomplete one, since it does not take account of all possible contingencies (fluctuations in the output gap). We demonstrate that if a verifiable measure of the structural deficit can be agreed upon, then the structural deficit rule will be superior to the actual deficit rule in welfare terms, and it will avoid a procyclical bias in policy.

In addition, we argue that rules based on the actual deficit may suffer from a "credibility trap" that has not previously been identified. One of the implications of including escape clauses in an actual deficit rule is that it will necessitate imposing a higher marginal penalty on fiscal deficits than would otherwise be the case. Higher marginal penalties are necessary, because fiscal authorities will anticipate that there is an exogenously determined probability that the escape clause will be triggered, and, as a result, even a high deficit will not be sanctioned. However, it is possible to suggest that the greater the magnitude of the penalties stipulated in any fiscal deficit contract, the higher the probability that the penalties may never actually be imposed.[4] This may be true for the same reason that promising lengthy imprisonment would be an ineffective tool to reduce parking violations; penalties that are disproportionate to a violation will lack credibility. We show that under an actual deficit rule, when the principal who is enforcing a deficit contract has imperfect credibility, then there may in fact be no equilibrium announced penalty for excessive deficits that would induce fiscal authorities to implement an optimal policy. Under these conditions, fiscal policy will have an expansionary bias and, as a consequence, monetary policy will, other things being equal, have a contractionary bias. Even in the absence of this "credibility trap", expansionary fiscal and contractionary monetary policies can result in our model if there is imperfect initial reputation and a principal over-estimates his own initial reputation when designing rules.

In addition to its normative implications about optimal design of fiscal rules, our model also provides positive predictions that may be useful for understanding recent events in Europe.

of the policy instrument as a function of relevant variables - an 'instrument rule'. For clarity, we shall refer to monetary and fiscal 'rules' as constraints imposed on policy authorities at an 'institutional design' stage. For monetary policy this would then lead to an 'implicit rule' for the instrument. See Svensson (1999) for discussion.

[3] The presence of the escape clause is, as we shall argue, an equilibrium feature of actual deficit rules.

[4] This is a point that has been previously emphasized by Drazen (2002).

Under an actual deficit rule with imperfect initial reputation we predict that there will be greater likelihood of an expansionary bias in fiscal policy and a contractionary bias in monetary policy. We also predict that an actual deficit rule will lead to a procyclical bias in fiscal policy.

2 A Basic Monetary-Fiscal Interaction Model

We begin with a stylized model of monetary-fiscal policy interactions that draws on Dixit and Lambertini (2001, 2003) and Bilbiie (2001 a, b). The model is used as a simple and tractable vehicle for investigating strategic interaction between monetary and fiscal policy, and hence the optimal design of rules restricting the behavior of the two policy authorities. Our analysis can be seen as dealing with the optimal design of fiscal rules under certain practically relevant informational problems, *given* a need for such rules. The model features real rigidities (in the form of monopolistic competition, e.g.) making the natural rate of output (employment) too low. It also features nominal rigidities, making monetary policy have real short-run effects. Fiscal policy can be employed to address the real distortion (e.g. by subsidising the monopolist), but we assume that due to distortionary taxation, it cannot close the output gap perfectly; for instance, the taxes used to finance the subsidy are distortionary (or have deadweight losses), as otherwise fiscal policy alone could be used to achieve the socially optimal level of output. As detailed below, this will result in strategic interaction between the monetary and fiscal policymaker. These features result in a reduced-form for aggregate supply as given in equation (1), where the natural level of output (under sticky prices and monopolistic competition) is normalised to zero.[5] Note that fiscal policy can stimulate output even if a fiscal expansion is anticipated. The variable ε is a favourable supply shock, observed when policy is set, and the variable u is a shock to output that is not observed at the time policy is set. The coefficient a depends non-linearly on the nature of the fiscal policy one considers: it could be negative for non-keynesian effects or positive for more standard cases.[6] Equation (2) determines the price level, which is influenced by fiscal as well as monetary policy, with the coefficient c capturing any such interactions, where the sign of c again depends on the nature

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[5] Such a 'neoclassical' aggregate supply function could result if a fraction of prices could not be adjusted in each period, while distortionary taxes make real marginal costs vary beyond variations in aggregate demand.

[6] Appendix A in Dixit and Lambertini 2001 and 2003 presents a detailed analysis of the microfounded model delivering the reduced forms here, and of the effects of various policy experiments.

of the fiscal policy one considers in the underlying economy (e.g. a supply-side policy would actually lead to a decrease in average prices).[7] The private sector forms rational expectations of both deficit and money supply as in (3), where the information set Ω_{-1} contains past values of macroeconomic variables as well as values of all coefficients and the distribution of shocks. This implies that the expected price level is fully determined.

$$y = af + b\left(\pi - \pi^e\right) + \varepsilon + u \tag{1}$$

$$\pi = m + cf + v \tag{2}$$

$$m^e = E\left[m \mid \Omega_{-1}\right] \equiv \int m d\Phi\left(\varepsilon, v\right), f^e = E\left[f \mid \Omega_{-1}\right] \equiv \int f d\Phi\left(\varepsilon, v\right) \tag{3}$$

Note that the model we consider is static (aside 'quasi-dynamics' introduced by expectations). This modelling choice has been made exclusively on grounds of tractability. Introducing dynamics would enrich the channels of interaction between the two policy authorities, but the point of this paper is that even focusing on the static interactions, there is scope for policy cooperation, or rather for institutional design to achieve this. Moreover, we then want to study how various informational imperfections will change the optimal design of institutions. These points can be made in the present version of the model, and they would be reinforced by dynamic interactions such as debt sustainability issues, determinacy questions, etc..[8]

We assume fiscal and monetary policymaking are decentralised, each of the authorities having preferences represented by the following period loss functions, which differ from the one derived from social welfare as described below. In order to focus on a case where rules limiting fiscal and monetary policy may be desirable, we assume that $y^F > \overline{y} = 0$, where it is important to note that y^F produces less than the socially optimal level of output, precisely because of distortionary taxation. Additionally, we assume $y^M \geq \overline{y}$, hence the preferred output

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[7]Fiscal policy could influence the price level and/or inflation for a variety of reasons: expected monetisation of deficits or accomodation of fiscal expansions (due at least to political pressures on the central bank); or if fiscal policy is non-Ricardian as in the Fiscal Theory of Price Level. See *i.a.* Sargent and Wallace 1981, Leeper 1991, Woodford 2002.

[8]An early paper studying dynamic interactions between monetary and fiscal policy as a dynamic game is Tabellini (1986).

level of the monetary authority could actually be equal to the natural one (as argued e.g. by Blinder 1997) or could be larger, reflecting pressures on the central bank to increase output above the natural rate. The price level targets are not restricted, although one could think of a setup where the monetary authority has been already optimally assigned a price level target.[9] This will turn out to be the case in equilibrium. The relative weight on output stabilisation is the same for both authorities. This simplifies the algebra with no impact on the analysis.

$$L^F = \frac{1}{2}\left[\lambda\left(y - y^F\right)^2 + \left(\pi - \pi^F\right)^2\right] \tag{4}$$

$$L^M = \frac{1}{2}\left[\lambda\left(y - y^M\right)^2 + \left(\pi - \pi^M\right)^2\right] \tag{5}$$

The timing of the policymaking game will differ throughout the paper depending on specific details, but will be encompassed by the following sequence of moves: (i) rules are imposed on policy authorities if not specified otherwise (ii) expectations are formed as described (iii) observable shocks hit the economy (iv) policies are chosen (v) the unobserved shock occurs, and finally (vi) an enforcement stage occurs where the principal chooses whether to enforce the rules imposed on the fiscal and monetary authorities at stage (i).

In this model the social welfare function is quadratic in output and the price level, with bliss points $y^*, \pi^*, (f^* = 0)$, where output is at its flex-price perfect competition level (hence there is no need for fiscal policy) and prices equal the average level of preset prices in the economy (see Woodford 1999, and Appendix B in Dixit and Lambertini 2001). Here we approximate these bliss points by y^c, π^c which we will refer to as the "cooperative" outcome.[10] To solve for the optimal policy choices we first take the case where at stage (i) there are no rules imposed on the two authorities, but the monetary and fiscal authorities can cooperate and commit with respect to the private sector, solving the following expression.

$$\min_{m,f,m^e,f^e} E[\alpha L^M + (1 - \alpha) L^F] \text{ s.t. (3)} \tag{6}$$

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[9]Note that in our model inflation and price level targeting are equivalent, due to the repeated nature of the problem and the absence of intertemporal links.

[10]By 'approximate' we mean this is only second best as it involves the use of the discretionary fiscal instrument. The first best would mean eliminating the real distortion, but we suppose that this is unfeasible given distortionary fiscal instruments.

The solution to this problem is detailed in Bilbiie (2001a, 2001b) and we reproduce it here. We abstract for the moment from the presence of unforeseen shocks u. Under these conditions, the optimal policy mix is given by the following two equations.

$$
\begin{aligned}
f^c &= \frac{1}{a}y^c - \frac{1}{a}\varepsilon \\
m^c &= \pi^c - \frac{c}{a}y^c + \frac{c}{a}\varepsilon - v
\end{aligned}
\tag{7}
$$

In these expressions y^c, π^c are now weighted averages of preferred levels of the two authorities, $y^c = \alpha y^M + (1-\alpha)y^F$ and $\pi^c = \alpha\pi^M + (1-\alpha)\pi^F$. Hence, the social loss would be approximated by (8).

$$
L^c = \frac{1}{2}\left[\lambda(y-y^c)^2 + (\pi-\pi^c)^2\right]
\tag{8}
$$

In this case as long as the two authorities can cooperate it does not matter whether they can commit in advance to their respective policies. Given cooperation, the cooperative equilibrium is already time consistent (for a full treatment see Bilbiie 2001a, 2001b). The relevant intertemporal welfare objective is $\Lambda^i = \sum_{t=0}^{\infty}\beta_i^t L_t^i$, where $i = M, F, c$.[11]

When we instead consider a case without cooperation (and no delegation for the moment), the equilibrium is Nash discretionary. At stage (iv) the authorities will solve: $\min_m L^M$ and $\min_f L^F$, both taking expectations and the other authority's strategy as given. Under these conditions, the policies chosen by each authority in equilibrium will be as follows.

$$
\begin{aligned}
f^n &= \frac{1}{a}y^n - \frac{1}{a}\varepsilon \\
m^n &= \pi^n - \frac{c}{a}y^n + \frac{c}{a}\varepsilon - v
\end{aligned}
\tag{9}
$$

These equilibrium policies are different from the optimal ones, since the values of output and price level arising are different from the optimal levels.[12] In general, when compared to the constrained optimum there will be a structural deficit bias, and monetary policy will be excessively contractionary, due to non-internalisation of policy externalities. However, there is no stabilisation bias, as responses to shocks are identical under the two policy equilibria

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[11] For most of the cases considered below, the dynamic programming solution to minimising the intertemporal loss boils down to period-by-period minimisation of the period loss function due to the assumed structure of the economy.

[12] Specifically, the Nash equilibrium values of output and inflation are: $y^n = y^F + \frac{c}{a\lambda}\left(\pi^F - \pi^M\right) + \frac{bc}{a}\left(y^F - y^M\right)$ and $\pi^n = \pi^M - \frac{bc}{a}\left(\pi^F - \pi^M\right) - \left(1 + \frac{bc}{a}\right)b\lambda\left(y^F - y^M\right)$.

considered. This is consistent with both empirical evidence and with recent theory (see for instance Canzoneri, Cumby and Diba, 2002 and Blinder's comment therein).

We now consider which rules or institutions set at stage (i) could achieve the optimal policy mix even if the two authorities do not cooperate (and do not commit with respect to the private sector). We consider the possibility that a common principal could influence policy by giving a price level target to the monetary authority and a linear deficit contract to the fiscal authority.[13] In this case, the two independent policy authorities will act discretionarily and non-cooperatively, minimising the following assigned loss functions (where superscript D stands for 'delegated', and 'o' for optimal).

$$
\begin{aligned}
L^{MD} &= \frac{1}{2}\left[\lambda\left(y - y^M\right)^2 + (\pi - \pi^o)^2\right] \\
L^{FD} &= L^F + T\left(., f\right) = L^F + \tau^o\left[f - f^c\right]
\end{aligned}
\tag{10}
$$

Once we assume that the principal can create policy rules of this type, the unique policy design mechanism (π^o, τ^o) is found in Proposition 1.

Proposition 1 *The unique optimal and subgame perfect policy design mechanism implementing the cooperative optimum in the non-cooperative game is:*

$$
\begin{aligned}
\pi^o &= \pi^M - b\lambda(y^M - y^c) \\
\tau^o &= (a + bc)\lambda\left(y^F - y^c\right) + c\left(\pi^F - \pi^c\right)
\end{aligned}
$$

Proof. See Appendix A. ■

In this context, there are no incentives to deviate *ex post* as long as the same principal delegates both policies, and shares L_c. Were it not for the 'common principal' assumption, the optimal mechanism would differ from the subgame perfect (credible) one, as separate principals would have incentives to deviate from optimal delegation, consistent with their initial non-cooperative incentives.[14]

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[13] The same could be achieved for other delegation schemes, but here we restrict choice to linear contracts for two reasons: technical convenience (this also insures uniqueness as described below) and practical relevance (as this resembles the SGP's structure).

[14] For an illustration of this argument in international monetary policy cooperation and delegation, see Bilbiie (2000).

Having found the optimal rules, we next want to look at issues related to their design when fiscal policy outcomes depend on automatic stabilizers and when credibility of enforcement of the rules is imperfect. In the case considered so far, it is subgame perfect for a common principal to sanction the monetary and fiscal authorities if they choose policies that deviate from π^o and f^c, but in practice there may be initial uncertainty about a principal's commitment to enforcing an announced rule. Because we are primarily concerned with analyzing fiscal policy rules, we assume throughout the remainder of the paper that monetary policy is delegated with the optimal price level target, and this target is credible.

3 Fiscal rules and automatic stabilizers

Consider the case where the unforeseen output shock u that hits at stage (v) has an impact on fiscal policy outcomes due to the presence of automatic stabilizers. This introduces a meaningful distinction between the actual fiscal deficit f and the structural deficit s. In expression (11) the coefficient γ captures the extent to which government spending and revenues are cyclically sensitive; we assume that this coefficient is beyond authorities' control in the short run.[15]

$$f = s - \gamma u \tag{11}$$

We shall in the remainder ignore the observable shocks ε, as we have seen these can be stabilised perfectly. Hence, we focus on u shocks. Let the distribution of these shocks be given by $G(u)$ over a support $\mathbf{U}$ (for example $\Re$, the set of real numbers), and for the moment assume the distribution is common knowledge to fiscal, monetary authorities and the private sector. We shall also focus, unless otherwise stated, on the fiscal policy rules and institutions, assuming that the monetary policy situation is clear, i.e. the price level targeting regime is credible and perfectly enforced. Now s is the fiscal policy instrument and its optimal value is found by solving $\min_{m,s,m^e,s^e} \int_u L^c(u) \, dG(u)$. Following the same method as above and using the

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[15] Actually, the formula would be $f = s - \delta y$, but to the extent we can then substitute f in the reduced forms and rewrite everything in terms of s, f will ultimately depend on the unforseen shock. The structural coefficients will then modify. This is a simplification but captures the main idea that there are cyclical fluctuations affecting the actual deficit in an intuitive and tractable way. This model specification follows Canzoneri, Cumby, and Diba (2002).

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fact that shocks are unobservable, the solution is expressed in (12) for the structural and then, implicitly, the actual deficit.

$$s^c = \frac{1}{a}y^c \rightarrow f^c = \frac{1}{a}y^c - \gamma u \tag{12}$$

Fiscal policy is optimally not employed for discretionary cyclical stabilisation, but shocks affect the actual deficit through automatic stabilisers. Hence, the optimum will be attained in expected terms. Had the shock been foreseeable, the optimal rules would be given by expression (13).

$$s^c(u) = \frac{1}{a}y^c - \frac{1}{a}u + \gamma u \rightarrow f^c(u) = \frac{1}{a}y^c - \frac{1}{a}u \tag{13}$$

This shows that the optimal value of the automatic stabiliser would be $\gamma = \frac{1}{a}$. In practice, this parameter will capture the extent to which a tax system is progressive and there are welfare benefits such as unemployment compensation. We assume this parameter is not controllable by the fiscal authority in the short run. The policies chosen by the fiscal authority under discretion at stage (iv) will now be a solution to $\min_s \int_u L^F(.) \, dG(u)$. [16]

$$s^n = \frac{1}{a}y^n, f^n = \frac{1}{a}y^n - \gamma u \tag{14}$$

Once again, there is a structural deficit bias. In the next sub-section we will consider the possibility that the principal might establish a contract based on the structural deficit, which could remove this bias. We will also consider the alternative of contracting on the actual deficit, even though it is known that fiscal policy responds to automatic stabilizers.

3.1 A complete contract: the structural deficit rule

Consider first a contract on the structural deficit: $T_s = \tau_s \left[s - \frac{1}{a}y^c \right]$. This would allow automatic stabilizers to work while still achieving the cooperative outcome with respect to fiscal policy. Drawing on the contract theory literature, this type of rule would be a *complete contract*, because it specifies all possible contingencies. In this case renegotiation is usually welfare-reducing (see Salanie 2000). As with any complete contract, this relies on the

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[16] Actually y^n, π^n is now the Nash equilibrium, but given optimal delegation to the central bank, which is of course closer to the optimum than the one without delegation, we can replace π^M with π^o in the relevant Nash equilibrium expressions.

contracted measure being *ex post* verifiable. In this case the question would involve the reliability of the measure of s, which itself depends upon the estimated $\hat{\gamma}$. Verifiability means that the principal not only observes s (and γ) but also is ready to accept it as the basis for a contract (for decision on enforcement).[17] This is obviously a key issue in the EMU context. Under these conditions, the optimal and credible contract is found to be the same as before, following exactly the same solution method. Under this rule, the fiscal authority solves $\min_s \int_u \left[L^F\left(.\right) + \tau_s s \right] dG\left(u\right)$ and the optimal marginal penalty is given by:

$$\tau_s^o = \tau^o = (a + bc)\lambda\left(y^F - y^c\right) + c\left(\pi^F - \pi^c\right) \tag{15}$$

This contract can be shown to be subgame perfect using the same solution method as in Appendix A. This rule (together with the price level target) implements the optimal policy mix in (12).

Even if one assumes that the measure of s is verifiable, there remains a potential problem of implementation errors with the structural deficit rule, because the principal may have an inaccurate estimate of γ.[18] As a result, the principal "could get it wrong", in the sense of penalizing the well-behaved fiscal authority or not penalising a fiscal authority that has actually run a structural deficit. The former outcome would happen for a given realisation of the unforeseen shock $\tilde{u}$ if $\hat{s} > s^c$, where $\hat{s}$ denotes the estimated structural deficit by regressing observed past deficits $\tilde{f}$ on observed past shocks $\tilde{u}$. Substituting in the 'penalising condition' we get the following expression (where values without $\sim$ or $\hat{}$ denote actual values).

$$\tilde{f} + \hat{\gamma}\tilde{u} > s^c \rightarrow s - \gamma\tilde{u} + \hat{\gamma}\tilde{u} > s^c \tag{16}$$

It is then clear that mistakes in enforcement occur as long as $(\hat{\gamma} - \gamma)\tilde{u} < 0$ is satisfied where γ is the 'true' cyclical sensitivity. This means overestimating $\hat{\gamma}$ for a negative shock and underestimating for a positive shock.

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[17] As a result, the argument that unverifiability might make an 'incomplete' contract necessary is different from an argument emphasizing the possibility of "unforeseen" or "indescribable" contingencies. Maskin (2001) provides a recent discussion of this distinction.

[18] This could be due to both limitations in data for estimating the structural deficit, as well as to the fact that the automatic stabilizer parameter γ will be changing over time (as emphasized by Blanchard and Perotti, 2002). Following the Lucas critique, it is also possible that the imposition of a fiscal rule would itself lead to a change in γ.
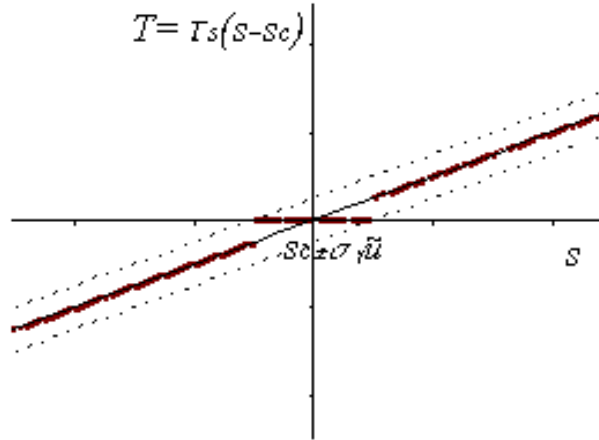
Figure 1: Error bands on a structural deficit target

One simple way to address this problem would be to attach error bands on the structural deficit target. The width of the error bands would depend on the magnitude of the standard errors of the $\hat{\gamma}$ coefficient. Figure 1 presents a graphical illustration of the argument. The upward sloping line passing through the origin plots the contract not taking into account uncertainty. The dotted parallel lines represent shifts in this introduced by uncertainty about γ. Assuming that the distribution of the estimated $\hat{\gamma}$ is such that 95% of outcomes lies within two standard deviations, $[\hat{\gamma} - \sigma_{\hat{\gamma}}, \hat{\gamma} + \sigma_{\hat{\gamma}}]$, we can view the two dotted lines as representing the bounds $T = \tau_s \left[\tilde{f} + \hat{\gamma}\tilde{u} \pm \sigma_{\hat{\gamma}}\tilde{u} - \frac{1}{a}y^c \right]$. Then, to minimise errors, the penalty would take a form like the thick solid line, not penalising for a measured structural deficit within some bounds from s^c, and then penalising according to the estimated $\hat{\gamma}$ outside these bounds. These can be seen as error bands on the structural deficit target, related to the standard deviation of the cyclical sensitivity but also to the realisation of the shock, of the form $s^c \pm \sigma_{\hat{\gamma}}\tilde{u}$.

3.2 An incomplete contract: the actual deficit rule

It might be the case that the structural deficit rule is infeasible because the structural deficit is not verifiable. This might mean that the contract can be written only on the actual deficit, taking the form $T_f = \tau_f \left[f - \frac{1}{a}y^c \right]$ for any realisation of the shock u. Under the actual deficit

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rule, the timing of the game (ignoring the monetary authority and the private sector, whose behaviour is unaltered) is as follows: first, the rule based on f is announced, then the structural deficit s is chosen by the fiscal authority, then the shock u is realised and hence f becomes observable, and finally the rule is enforced (or not). In the language of contract theory this rule would be called an incomplete contract, because it leaves the *ex post* observed structural deficit uncontracted since it is unverifiable. In such cases renegotiation can be welfare-improving (see Salanie 2000, e.g.). Note that here contract incompleteness comes from unverifiability of the structural deficit (or the cyclical sensitivity γ), and not from unboservability, bounded rationality or indescribability, as is the case in part of the incomplete contracts literature. In fact, applying this rule strictly would mean penalizing a 'well-behaved' fiscal authority for adverse shocks. The intuition is as follows. Consider a fiscal authority that respects a rule, choosing $s = \frac{1}{a}y^c$. For a given realisation of $\tilde{u}$ its penalty will be as follows: $T_f = \tau_f\left[-\gamma.\tilde{u}\right]$. This means it will be sanctioned for an adverse shock and rewarded for a favourable shock. Under these circumstances, the principal will always commit an error in implementing the rule, and the magnitude of this error will be given by the cyclical sensitivity. One can immediately see here that such a rule produces the opposite effect of automatic stabilization.

The problem with the actual deficit rule is that it generates a procyclical bias in the discretionary part of fiscal policy. Consider the case where the fiscal authority is able to forecast the shock u, and it has an expectation about the distribution of the shock $G^F(u)$, with the expectation also satisfying $E^F(u) = \int u dG^F(u) \neq 0$. Under these conditions, the fiscal authority will choose the structural deficit such that *in expected terms* (expectations with respect to $G^F(u)$) it fulfills the actual deficit rule $\int f(u) dG^F(u) = \frac{1}{a}y^c$. This implies choosing $s = \frac{1}{a}y^c + \gamma \int u dG^F(u)$. This will lead to the choice of a higher than optimal structural deficit when a positive shock is expected and a lower one otherwise, and this outcome is clearly suboptimal (being the opposite of stabilisation). The rule thus induces a procyclical bias, and were the shock perfectly foreseeable to the fiscal authority (but not to the principal at the time the contract is written), the perverse effect of this rule would be to cancel automatic stabilisation completely. Imposing a state-independent deficit limit $f \leq \bar{f}$ (much like the SGP does), where $\bar{f} > \frac{1}{a}y^c$, does not solve this problem. It would would still penalise incorrectly for more severe shocks, it would fail to penalise in 'good times', and as a result, it would still induce a procyclical bias.

As s is considered unverifiable, full ex-post flexibility in applying the actual deficit rule is not possible (this would actually mean that at the enforcement stage the principal relies upon γ). In order to make the rule 'more complete', the principal can impose the rule with an *escape clause*. This rule would specify that for some values of the shock inside the interval $[\underline{u}, \overline{u}]$ the rule will be applied, while for some extreme values outside it will not, and this is specified ex ante. This is similar to the current design of the EU Stability Pact. This means that the contract is made more complete. However, this modification comes at a cost. If the fiscal authority knows there is some exogenous probability that an escape clause may be triggered, then, intuitively, the introduction of this possibility will influence the deficit the fiscal authority chooses in equilibrium, shifting the best response towards choosing a higher structural deficit. As a consequence, we will show that if a rule contains an escape clause, then, if it is to induce the fiscal authority to choose a policy in equilibrium that is consistent with the cooperative outcome, such a rule will have to include a higher marginal penalty on deficits than would be the case for a rule without an escape clause. The marginal penalty will have to be modified as shown below. At stage (iv) the fiscal authority will face the following additional term in its loss function L^F, taking $\tau_f, \underline{u}, \overline{u}$ as given and solving for $s(\tau_f, \underline{u}, \overline{u}\,)$.

$$T_f = \begin{cases} \tau_f \left[f - \frac{1}{a} y^c \right], \text{if } u \in [\underline{u}, \overline{u}] \\ 0, \ u \notin [\underline{u}, \overline{u}] \end{cases} \tag{17}$$

The expected loss function under this rule will then be (where the integral is taken over the whole support of u when not specified otherwise):

$$E\left[L^F \right] = \int L^F \left(. \right) dG \left(u \right) + \int_{\underline{u}}^{\overline{u}} \tau_f \left[f \left(u \right) - \frac{1}{a} y^c \right] dG \left(u \right) \tag{18}$$

Minimisation of this with respect to s, assuming throughout optimal and credible monetary delegation and rational expectations leads to the following instrument rule (the derivation again follows the method used above):

$$s(\tau_f, \underline{u}, \bar{u}) = \frac{1}{a} y^n - \frac{1}{a^2 \lambda} \tau_f \left[G \left(\bar{u} \right) - G \left(\underline{u} \right) \right], G \left(\underline{u} \right) = \int_{-\infty}^{\underline{u}} dG \left(u \right), G \left(\bar{u} \right) = \int_{-\infty}^{\bar{u}} dG \left(u \right) \tag{19}$$

Greater *ex ante* flexibility (the smaller the $[\underline{u}, \overline{u}]$ interval) and smaller penalties imply that the equilibrium gets closer to y^n. Less flexibility and higher marginal penalties would lead to the optimum. Now, looking at the choice of subgame perfect contract in all components (marginal penalty and thresholds), we see they cannot be pinned down individually. Formally, at stage (i) the principal chooses τ_f and escape clause $\underline{u}, \overline{u}, (finite)$ to maximise social welfare taking

into account the best response of fiscal authority (18) and the counterpart for the monetary authority. For details of the proof see Appendix B. There is in fact an infinity of equilibria, consisting of the schedule:

$$\tau_f = \frac{\tau^o}{G(\bar{u}) - G(\underline{u})} = \frac{\tau^o}{\int_{\underline{u}}^{\bar{u}} dG(\underline{u})} > \tau^o, \underline{u} \neq \bar{u} \in int(\mathbf{U}) \tag{20}$$

This is a first indeterminacy result: all penalties and thresholds satisfying (20) are equivalent for the principal in terms of welfare, and they are all subgame perfect. The choice of the escape clause is bound to be arbitrary, due to the very reason that made the complete contract impossible: unverifiability of γ. An optimal rule with an escape clause could be found, but that would be equivalent to writing the contract on the structural deficit (that would mean making the deficit target state-contingent, and this would bring us back to the complete contract). Note that under this rule, the procyclical bias identified before is still present, although of a lower magnitude. Under forecasting (strategic use of private information) of the shock, the best response of the fiscal authority is now $s(.) = \frac{1}{a}y^c + \gamma \int_{\underline{u}}^{\bar{u}} u dG^F(u) = \frac{1}{a}y^c + E^F[u \mid u \in [\underline{u}, \bar{u}]]$, where the expectation is conditional upon the shock being inside the interval where the penalty is binding. Existence of this bias is in fact a feature of this rule, no matter how 'incomplete' the contract actually is, although its size does vary with length of the $[\underline{u}, \bar{u}]$ interval. The more flexibility allowed *ex ante*, the smaller is the procyclical bias (recall this was one reason to consider escape clauses in the first place). Note also that the marginal penalty under the rule with an escape clause is always larger than the penalty for the structural deficit case, and more so the greater the flexibility allowed *ex ante* (the smaller the $[\underline{u}, \bar{u}]$ interval). Indeed, as the interval collapses, the procyclical bias is eliminated but the marginal penalty tends to infinity, $\lim_{\underline{u} \to \bar{u}} \tau_f = \infty$. This is an extreme, but illustrative example of the tradeoff faced by the principal in designing an actual deficit rule.

3.3 Summary

In this section we have demonstrated how a linear contract written on the structural deficit can produce an optimal outcome regarding fiscal policy, provided that the estimate of the structural deficit is considered verifiable. Under such a rule, implementation errors will be minimised by including error bands on the deficit target, proportionate to uncertainty about the estimate of the structural deficit. For the case of contracts written on the actual deficit, expected

welfare can be improved by including an escape clause, but such contracts will still generate a procyclical bias in fiscal policy. In the extreme case, under such a rule the procyclical bias that is induced would, in fact, perfectly offset the action of any automatic stabilizers. Finally, we have also shown that incorporation of escape clauses in rules written on the actual deficit will require the introduction of higher marginal penalties. We will argue in the next section that this might ultimately undermine the credibility of such rules.

4   Credibility, Reputation and Enforcement

So far we have looked at policy design mechanisms that are optimal and subgame perfect if the principal shares the social welfare function L^c. With this setup, enforcement of fiscal and monetary policy rules is fully credible and fully anticipated. In practice, however, there may be initial uncertainty whether an announced set of rules will actually be enforced. This section considers this problem explicitly. We first consider a case where initial reputation is imperfect and the principal can make a costless announcement in the first stage regarding the penalty that will be applied to fiscal deficits. We then consider a case where these initial announcements are "costly" in the sense that the announcement of a larger penalty leads to a greater expectation that the penalty may actually never be applied.

In order to speak meaningfully about reputation, a source of incomplete-information has to be introduced, which in our case shall translate to uncertainty about the principal's type. The intuition is that the public and the two policy authorities (notably, the fiscal authority) do not know, when the rule is announced, whether the principal will actually enforce it *ex post*. Suppose for simplicity that the prior common beliefs of the private sector and policy authorities about the principal's types are that he is either T(ough) with probability ω or W(eak) with $1 - \omega$. The principal's type will hence be described by its ability to follow up on commitments, as in Cukierman and Liviatan's (1991) and Barro's (1986) study of central bank policy under incomplete information, and not by differences in the objective function, although one could see the latter as a particular case (or an implication) of the former. In the cases considered up to now, commitment to the optimal penalty is perfectly credible - the policymaker announces it, and then is known to implement it with probability one, which in turn ensures implementation of the desired equilibrium in the monetary-fiscal policy game.

With imperfect initial reputation however, the announcement made by the principal has only partial effect on both expectations and strategies of the monetary and fiscal authority.

We assume that the tough policymaker T will announce a contract and at stage (vi) will actually enforce it. The weak policymaker W will announce a contract but will not enforce it. This could be because the weak type is more subject to political pressures from the fiscal authority, or for some other plausible reason that lies beyond the scope of our model. With imperfect information the main intuition is easy to grasp from the one-period model. We look for the endogenous announcement τ^A that would be needed to implement the optimum.

4.1 Reputation and enforcement with costless announcements

4.1.1 The structural deficit rule

We shall for the moment concentrate on the complete contract $T_s = \tau_s \left[s - \frac{1}{a} y^c\right]$. The fiscal and monetary authorities, and the private sector, know that the principal can be of one of the two types, but do not know which is in office. Consider first the optimal announcement for the tough principal (if he chooses to make an announcement, which shall be the case in equilibrium). Now the fiscal authority (sharing the prior about principal's type ω with the public and monetary authority) faces the following expected loss, after receiving the announcement.

$$E_\omega \left[L^F\right] = \int L^F \left(u\right) dG \left(u\right) + \omega \tau_s \left[s - \frac{1}{a} y^c\right] + \left(1 - \omega\right) 0 \tag{21}$$

The Nash equilibrium in the monetary-fiscal game leads to the expected values of output and price level as functions of the announcement:

$$\int_u y = y^c + \frac{1}{a\lambda} \left(\tau^o - \omega \tau_s^A\right), \int_u \pi = \pi^c - \frac{b}{a} \left(\tau^o - \omega \tau_s^A\right) \tag{22}$$

Plugging these back into $L^c\left(.\right)$ and minimising with respect to τ^A,and imposing $\tau^A = \tau$ (as the policymaker is dependable) leads to the value of the optimal announcement for the tough policymaker given below. There will be in fact an infinity of such announcements, i.e. an infinity of self-fulfilling equilibria and beliefs, but all other equilibria can be eliminated by using the 'intuitive criterion' of Cho and Kreps as is usually done in signaling games. This results in a unique equilibrium announcement.

$$\tau^A = \frac{\tau^o}{\omega} > \tau^o \tag{23}$$

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Under this announcement, the value of the social loss is zero, as the optimum is implemented. Without an announcement, the occurring equilibrium is the Nash equilibrium, and the loss is positive, hence the tough policymaker will always strictly prefer to make an announcement. The weak policymaker would find it optimal to choose the same announcement, as otherwise he would be revealed as weak and the fiscal authority would choose its instrument correspondingly. Then, at stage (vi) T enforces the contract, W does not. Hence, a tough principal has to play tougher in order to be perceived as such (to signal its type) if its initial reputation is low.[19]

We can now have a first glance at the macroeconomic effects of incomplete information about the principal's type when the principal sticks to the optimal announcement under complete information.[20] Specifically, this is done by considering $\tau^A = \tau^o$ above and then solving for the best responses of policy authorities to the announcement in equilibrium.

$$
\begin{aligned}
s\left(\omega, \tau^o\right) &= \frac{1}{a} y^c + \frac{1}{a^2 \lambda}\left(1 - \omega\right) \tau^o > s^c \\
m\left(\omega, \tau^o\right) &= \pi^c - \frac{c}{a} y^c - \frac{c}{a^2 \lambda}\left(1 - \omega\right) \tau^o < m^c
\end{aligned}
\tag{24}
$$

If the announcement were τ^o, the resulting policy mix would have a deficit bias for fiscal policy and a contractionary bias for monetary policy, both of which would be more severe as $\omega \to 0$. Hence, if the principal overestimates his own initial reputation, this can have perverse effects on both policy authorities, and these effects can be contrary to those the rule was supposed to achieve in the first place.

The above case concerns a one-period setup, which is enough for grasping the main intuition. In a two (or multi–)period setup, a new element arises in that first-period actions reveal information about the type of the principal, hence in choosing his first-period announcement and in deciding whether to enforce, the principal has to take into account these effects. In general, this leads to separating, pooling and mixed-strategy equilibria (see Cukierman and Liviatan (1991), Backus and Driffill (1986) or Vickers (1986) for applications to monetary

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[19] The result is the opposite of what Cukierman and Liviatan get for monetary policy: in their model, the dependable central bank has to accommodate expectations and play weaker (i.e. announce a higher inflation rate than the complete-information optimal one) when it has a lower initial reputation. Our result is more similar to Backus and Driffill (1985) and Vickers (1986), where the tough central bank has to play even tougher in order to signal its type.

[20] While this is behaviour off the equilibrium path in the model analysed here, it might illustrate a case whereby the principal overestimates her own initial reputation ω.

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policy games). In our case however, it will turn out that only pooling equilibria exist as long as we regard the two types as sharing the social loss function L^c. This is intuitive as there will always be a benefit for the weak type in mimicking the strong policymaker. If instead one described the weak type as having a different loss function, e.g. $L^F(.)$, then separating equilibria would exist. The intuition being straightforward, we do not present a detailed proof of this result, but the case we consider is, in our view, a more realistic one. The bottom line is that in general, the mere presence of uncertainty about the principal's type makes the optimal announced penalty higher. Should this not be recognised by the principal at the institutional design stage, a contractionary monetary and an expansionary fiscal policy bias would result.

4.1.2 The actual deficit rule

Consider now the incomplete contract which is written on the actual deficit $T_f = \tau_f \left[f - \frac{1}{a} y^c \right]$. Repeating the same exercise as before, the expected loss of the fiscal authority is given by the following expression.

$$E_\omega \left[L^F \right] = \int_u L^F(u) \, dG(u) + \omega \int_{\underline{u}}^{\bar{u}} \tau_f \left[f(u) - \frac{1}{a} y^c \right] dG(u) + (1 - \omega) \, 0 \qquad (25)$$

One obtains the optimal announcement by the same method. Note that now the threshold levels are themselves part of the announcement, hence the decision variables of the principal are $\tau_f^A, \bar{u}^A, \underline{u}^A$. The optimal announcement satisfies the inequality in (27).[21]

$$\tau_f^A = \frac{\tau^o}{\omega \left[G(\bar{u}^A) - G(\underline{u}^A) \right]} > \tau^A > \tau^o \text{ if } \bar{u}^A \neq \underline{u}^A \qquad (26)$$

There is again an indeterminacy of equilibria - the marginal penalty and the escape clause cannot be pinned down individually. Note that the inequality in (26) implies that under incomplete information, the optimal marginal penalty under an actual deficit rule is again larger than in the case of the rule written on the structural deficit. The penalty is also higher than the optimal penalty with complete information (being equal only for perfect initial reputation $\omega = 1$).

The various equilibria are pictured in Figure 2 which plots the optimal announced penalty τ^A on the horizontal axis against the $\tau^A \omega \left[G(\bar{u}^A) - G(\underline{u}^A) \right]$ function on the vertical axis. The

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[21] The proof of Proposition 3, for a more complicated case, contains a proof of this statement as a special case. See the next subsection.
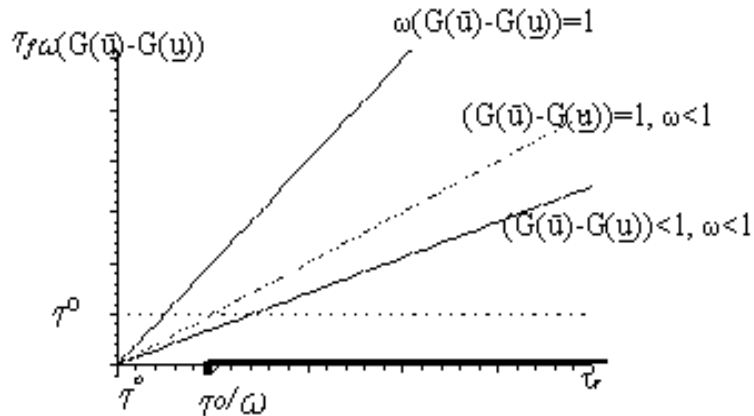
Figure 2: Optimal announcements under imperfect reputation

first intersection of the 45 degree line with the horizontal line would be the optimal contract when there is a perfect initial reputation. The second would be the structural deficit rule under imperfect reputation, and any point thereafter on the thick portion of the x-axis can be an equilibrium depending on the interval for threshold levels of shocks. The third line provides an example of the announced penalty under an actual deficit rule under imperfect reputation and where there is an escape clause $G\left(\bar{u}^A\right) - G\left(\underline{u}^A\right) < 1$. When an escape clause is included the same discussion as in the end of the previous section applies: the smaller the interval on which the rule is applied, the further the optimal announcement τ^A is from τ^o.

4.2 Reputation and enforcement with costly announcements

At first glance, it would seem from the previous section that incomplete information has little effect on the outcome of the monetary-fiscal game we consider; the problem of an imperfect initial reputation, if fully recognised at the institutional design stage, can be easily solved by announcing a higher marginal penalty than would otherwise be the case. Some observers have made this argument with regard to the EU Stability Pact. If there is uncertainty whether the Pact will be enforced, then why not deal with this problem by simply announcing larger penalties in order to ensure compliance?[22] What this solution overlooks is that as announced

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[22]See the *Financial Times*, October 24th 2002.

penalties grow larger, it may become increasingly implausible that they will ever be applied.[23] To take a banal example, one could attempt to ensure 100% compliance with parking restrictions by announcing penalties of lengthy imprisonment for even the most minimal parking violations, but, in equilibrium, such announcements might have little effect, because the public would anticipate that such severe sanctions would never actually be applied. The exact reason the sanctions would never be applied is an issue that goes beyond the scope of this paper, but it clearly involves the extent to which a sanction is seen as being 'disproportionate' to a violation. It is important to emphasize that this involves a different reason for believing sanctions may not be applied than would be the case when a principal has imperfect initial reputation ($\omega < 1$). What this situation implies is that policymakers considering announcing a higher marginal penalty on fiscal deficits will face a tradeoff. On one hand, to the extent the fiscal authority believes the sanction will be applied, a larger sanction will have a greater influence on its behavior. On the other hand, announcing a larger sanction may make it less believable that the sanction would ever actually be applied. We will show in this section that this problem can lead to a "credibility trap". For a policymaker who initially lacks a reputation for enforcement, there may be no equilibrium announcement that will ensure application of the optimal fiscal policy. We also show that while this problem exists for a contract written on the actual deficit, it does not emerge for a contract written on the structural deficit, precisely because the latter requires smaller marginal penalties and the former has to include escape clauses.

Consider the same setup as before (focusing on the one-period case), modified in one important respect: the higher the announced marginal penalty τ^A, the lower is the probability that it will be enforced. We adopt this assumption as a way of introducing the possibility that large marginal penalties may not be enforceable if they appear disproportionate to the violations caused. We model this enforcement probability independently from the probability of the policymaker being 'tough', ω, which is exogenous. Hence for any announcement τ^A, there will be a value $e\left(\tau^A\right)$ defined as $e\left(\tau^A\right) : \Re_+ \rightarrow [0.1]$ describing the probability that this penalty will be enforced. For the moment we assume that $e\left(0\right) = 1$ (announcing 'no penalty' is perfectly credible), $e\left(\tau^A\right) \leq 0$ (which captures the idea mentioned above) and there exists a value $\bar{\tau} > \tau^0 > 0$ such that $e\left(\tau^A\right) = 0$ for any $\tau^A > \bar{\tau}$. The last assumption shows there

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[23] This is closely related to an argument recently made by Drazen (2002).

is a maximal level of the penalty to which a positive enforcement probability is attached.[24] We assume the optimal penalty under complete information has some positive enforcement probability. The function $e\left(.\right)$ is assumed to be continuous and differentiable on $[0, \bar{\tau}]$. We make no further assumptions regarding the functional form of $(.)$ at this stage.

For simplicity, we shall focus on the case where $\tau^A > 0$, i.e. for penalties. As the function $e(x)$ satisfies $e(x) = e(-x)$, then $xe(x) = -xe(-x)$, the analysis for rewards will perfectly mirror the analysis for penalties.

Note that the function $e(.)$ is treated for the moment as given - it will turn out that its form and properties are crucial for the existence and properties of equilibria. In Figure 3, which graphs the probability of enforcement $e\left(\tau^A\right)$ on the y-axis against the announced penalty τ^A on the x-axis, some plausible forms of $e(.)$ are presented. The thick solid line at the top presents an ideal, benchmark situation in which any announcement less than $\bar{\tau}$ is perfectly credible. The dashed concave line corresponds to a situation where announcements are still somehow credible, as a concave function implies that the likelihood of enforcement decreases, but slowly. Finally, the convex function (solid line) represents a larger credibility problem, where the likelihood of enforcement decreases rapidly (and more so, the more convex the function is) even for low values of the announced penalty.

4.2.1 The structural deficit rule under costly announcements

We first consider how the structural deficit rule is affected by the possibility that announcements may be 'costly' in the sense that larger announced penalties may lead to diminished beliefs that penalties will be applied. The loss function minimised by the fiscal authority at stage (iv) for the structural deficit rule will be the following.

$$E_{\omega, e(.)}\left[L^F\right] = \int L^F\left(u\right) dG\left(u\right) + \int \omega \tau_s^A e\left(\tau_s^A\right)\left[s - \frac{1}{a}y^c\right] + (1 - \omega) 0 \qquad (27)$$

---

[24] Technically, we could have modelled this idea by assuming directly that the enforcement probability ω is a function of the announcement with the properties outlined above for $e\left(.\right)$. However, the two variables capture different phenomena. ω is the prior attached by private sector and policy authorities to the principal being tough, which in turn might depend on country characteristics (such as bargaining power of a country within the union to which the rule applies). In contrast, $e\left(.\right)$ is in turn a structural feature, depending on how disproportionate the penalty is with respect to any violation.
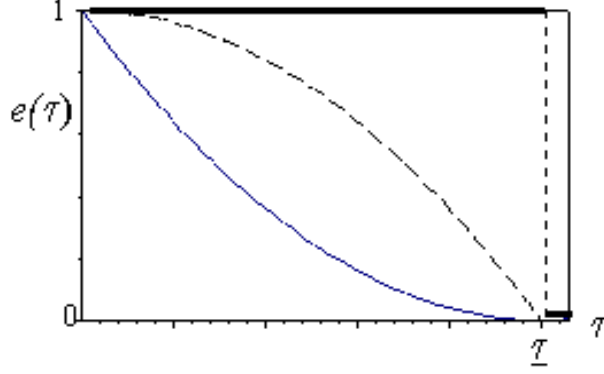
Figure 3: Enforcement probability function - possible forms

Using the same method as before, one finds the optimal subgame perfect contract announced by the principal. Proposition 2 emphasises some properties of the optimal announced penalty.

Proposition 2 *In the presence of incomplete information about the principal's type and credibility costs of announcements, the optimal subgame perfect marginal penalty on excessive structural deficits is given by:*

$$I \quad : \quad \tau_s^{*I} \in \left\{ \tau_s^A \ s.t. \ \tau_s^A e\left(\tau_s^A\right) = \frac{\tau^o}{\omega} \right\} \ if \max_{\tau_s^A} \tau_s^A e\left(\tau_s^A\right) > \frac{\tau^o}{\omega}$$

$$II \quad : \tau_s^{*II} \in \arg\max \left[\tau_s^A e\left(\tau_s^A\right) \right], \ if \max_{\tau_s^A} \tau_s^A e\left(\tau_s^A\right) < \frac{\tau^o}{\omega}$$

Proof. See Appendix B. ∎

Intuitively, what Proposition 2 says is that different equilibria can occur depending on a few parameters. First, if initial reputation is very low (hence $\frac{\tau^o}{\omega}$ is high) and/or if $e(.)$ is not very concave then case II may apply, a '*non-fundamental*' equilibrium. Here, the optimal announcement no longer depends on the structure of the economy. For the principal, the problem of minimising social loss by choosing the announced penalty is isomorphic to solving $\arg\max \left[\tau_s^A e\left(\tau_s^A\right) \right]$. In this case the credibility problem is so acute that the best the principal can do in terms of announcing a penalty is to balance its size and the likelihood of it being enforced as perceived by the public and policy authorities. As a consequence of this, there are in fact conditions under which the equilibrium announcement is even lower than τ^o, namely when $[\tau^o e\left(\tau^o\right)]\prime < 0$. Note that the equilibrium is self-enforcing by construction.
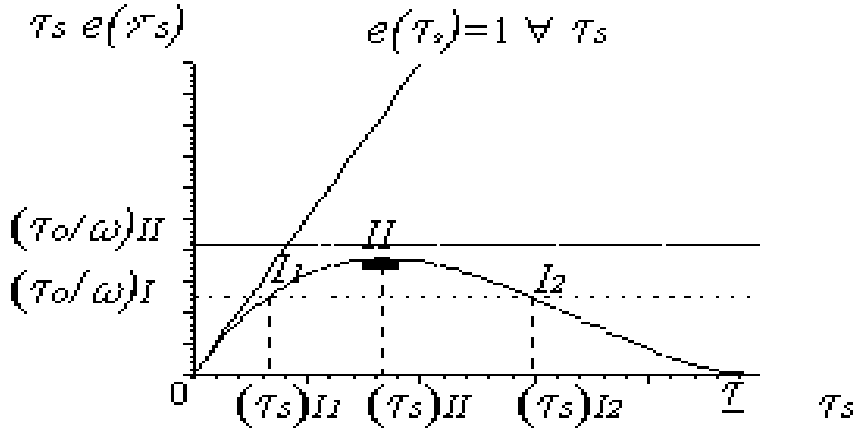
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Figure 4: Equilibrium announcement(s) for structural deficit rule

On the other hand, in case I there are *multiple equilibria*: there can be at least two equilibrium announcements with the same welfare properties. Focusing on the case where there are two equilibria (which is the case under plausible assumptions on the form of $e(.)$), one equilibrium would be characterised by a lower marginal penalty but higher enforcement probability, whereas the other equilibrium features a higher marginal penalty and lower likelihood of being enforced.

Figure 4 summarizes the results of Proposition 2, plotting the announced penalty multiplied by the probability of enforcement $\tau_s^A e\left(\tau_s^A\right)$ on the y-axis against the announced penalty on the x-axis. The two horizontal lines correspond to values of ω leading to the two different cases respectively, and the curve represents the $\tau_s^A e\left(\tau_s^A\right)$ function, which is equal to the 45 degree line when announcements are costless (illustrating the imperfect information case studied previously). The function collapses to the horizontal line when the penalty becomes so large that the probability that it will be enforced $e(.)$ becomes zero. If case I applies, the two equilibria are given by the points I1 and I2, whereas for case II (low ω or less concave or even convex $e(.)$) the equilibrium is given by II (note that in the figure we plot the case where ω is low, but this is equivalent to conditions on $e(.)$ being met such that case II applies).

Note that in Case I, the optimal announcement is always strictly larger than the optimal announcement with symmetric information, and it is also larger than the optimal costless announcement under imperfect initial reputation. This would lead to an even larger bias in the policy mix towards an expansionary fiscal policy and a contractionary monetary stance if

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the principal stuck to the optimal announcement under symmetric information τ^o. Generally, the biases would be given by the following two expressions (as calculated as in the previous section).

$$
\begin{aligned}
s\left(\omega, e\left(.\right), \tau^o\right) &= \frac{1}{a} y^c + \frac{1}{a^2 \lambda}\left(1 - \omega e\left(\tau^o\right)\right) \tau^o > s^c \\
m\left(\omega, e\left(.\right), \tau^o\right) &= \pi^c - \frac{c}{a} y^c - \frac{c}{a^2 \lambda}\left(1 - \omega e\left(\tau^o\right)\right) \tau^o \ll m^c
\end{aligned}
\tag{28}
$$

The expansionary fiscal bias is larger the lower is initial reputation and the lower is credibility for the given fiscal authority. Similarly, there is a contractionary bias of monetary policy which is also increasing in the same factor. One important thing to note is that these biases occur in the non-fundamental equilibrium *without the principal overestimating her initial reputation.*

A further point to note is that with imperfect credibility of the announcement and non-cooperative playing of the two authorities, once the 'optimal' (under perfect information) penalty τ^o is announced, the monetary authority is forced to accommodate, even if it is following the optimal targeting regime it has been assigned and there is no development in the economy asking for contractionary policy. One can view this as a main implication of imperfect credibility or enforcement of fiscal rules on the monetary authority. This has clear implications for current discussions about monetary and fiscal policy interactions in EMU.

4.2.2 The actual deficit rule under costly announcements

Consider now the rule on the actual deficit, taking into account as before the necessity of specifying an escape clause *ex ante* and looking for the optimal announcement in this case. As we examine only penalties for reasons mentioned in the beginning of the section, we can now consider only a lower threshold value/escape clause for the shock, i.e. a maximum negative shock $\underline{u}$.[25] Moreover, we shall only look at cases where $\underline{u}$ is a 'plausible' value, i.e. $G\left(\underline{u}\right) > 0$. Now the fiscal authority faces the following loss function.

$$
E_{\omega, e(.), u}\left[L^F\right] = \int L^F dG\left(u\right) + \int_{\underline{u}}^{\infty} \omega \tau_f^A e\left(\tau_f^A\right)\left[f - \frac{1}{a} y^c\right] dG\left(u\right) + \left(1 - \omega\right) 0
\tag{29}
$$

At stage (i), the principal announces $\tau_f^A, \underline{u}^A$ to minimise the social loss taking into account the reaction of the two authorities and rational expectations of the private sector, where the

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[25] One can think of the positive maximum shock as being the case $G\left(\bar{u}^A\right) = 1$.
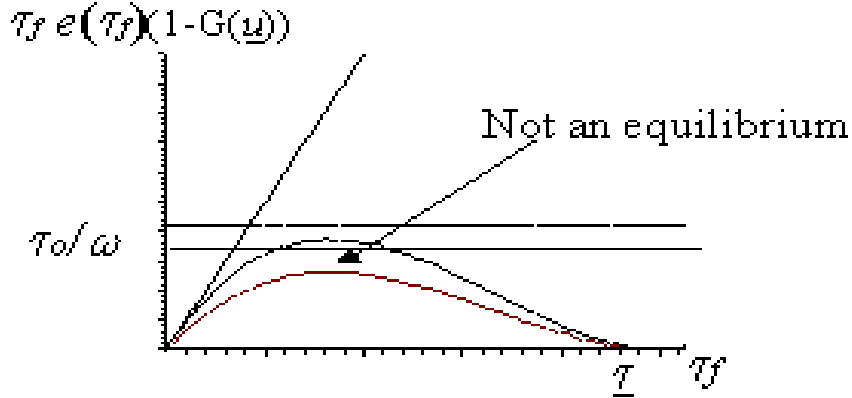
Figure 5: Equilibrium announcement(s) for actual deficit rule.

fiscal authority minimises the loss function above. Proposition 3 presents some properties of the optimal announcement for this case and a proof is provided in Appendix 3.

Proposition 3 *"The Credibility Trap". In the presence of incomplete information about the principal's type and credibility costs of announcements, the optimal subgame perfect marginal penalty on the excessive actual deficit τ_f^A and the escape clause $\underline{u}^A$ is given by (assuming $G'\left(\underline{u}^A\right) > 0$):*

$$I.\ \tau_f^{*I}, \underline{u}^{*I} \in \left\{ \tau_f^A, \underline{u}^A \ \ s.t.\ \ \tau_f^A e\left(\tau_f^A\right) = \frac{\tau^o}{\omega\left[1 - G\left(\underline{u}^A\right)\right]} \right\}\ if\ \max_{\tau_f^A} \tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right] > \frac{\tau^o}{\omega}$$

$$II.\ No\ equilibrium\ announcement\ exists\ if\ \max \tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right] < \frac{\tau^o}{\omega},$$

Figure 5 presents a graphical illustration of Proposition 3. Note that the thin solid line is the $\tau^A e\left(\tau^A\right)$ line from Figure 4. We therefore assume that $e(.)$ does not change functional form depending on the type of rule in place. However, the relevant function for studying equilibria does change, as this will be multiplied by the $(1 - G(\underline{u}))$ factor (which is less than one), inducing a contraction of the function on the $[0, \bar{\tau}]$ interval. Consider first the case of a 'high ω' represented by the lower horizontal line. In case I, the relevant function does not change significantly. The result is much different for Case II which is pictured in Figure 5 by the lower curve. In Case II no equilibrium announcement exists. To see the difference between the two cases consider first a 'low ω' scenario (upper horizontal straight line). Under

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this scenario with a structural deficit rule Case II of Proposition 2 applies, and the equilibrium is the 'non-fundamental' one. For the actual deficit rule, however (lower curve, or any curve below the original one) no equilibrium exists - that is Case II in Proposition 3 applies. The 'non-fundamental' equilibrium featured in Case II of Proposition II would only be an equilibrium here if the rule were specified without an escape clause ($G\left(\underline{u}^A\right) = 0$), which is a case we rule out, because it induces perverse incentives on the side of the fiscal authority as discussed in Section 3. The closer the announced threshold value of the shock is to zero, the flatter is the graph of the function.

Some of the results above reinforce the intuition from the previous example. In Case I, it is again the case that there can be at least two equilibria, just as in Proposition 2. But note that now the announced marginal penalty is higher (roughly by an order of $\frac{1}{1-G(\underline{u}^A)}$) than in the structural deficit case. One other important difference as compared to previous cases is that now it is more likely that Case II will apply - obviously, it is harder, ceteris paribus for $\max_{\tau_f^A}\tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right] > \frac{\tau^o}{\omega}$ to be fulfilled given that the function $\tau_f^A e\left(\tau_f^A\right)$ is now scaled down by a factor of $1 - G\left(\underline{u}^A\right)$.

The main insight, which we call a 'credibility trap', is that with a contract on the actual deficit and imperfect initial reputation, one could end up in a situation whereby there is no equilibrium announcement that could lead to implementation of the optimal policy mix. On one hand, the probability of ending up in Case II is higher under the actual deficit rule, and, on the other hand, ending up in Case II now implies that there is no contract announcement $\left(\tau_f^A, \underline{u}^A\right)$ that would be believed. This could happen for any (or a combination) of: small ω, fastly decreasing $e(.)$, and the closer $\underline{u}^A$ is to zero. Announcing any deficit penalty results in an extreme form of the bias, namely the policy instruments are set as in the Nash equilibrium. This outcome is equivalent in welfare terms to having no rule.

In Case I it remains true that sticking to an announcement that would be optimal were these imperfections not present results in biases in the policy mix similar to the ones we mentioned before. The biases are now larger than for the structural deficit case.

$$
\begin{aligned}
s\left(\omega, e\left(.\right), \tau^o, \underline{u}^A\right) &= \frac{1}{a}y^c + \frac{1}{a^2\lambda}\left\{1 - \omega e\left(\tau^o\right)\left[1 - G\left(\underline{u}^A\right)\right]\right\}\tau^o > s^c \\
m\left(\omega, e\left(.\right), \tau^o, \underline{u}^A\right) &= \pi^c - \frac{c}{a}y^c - \frac{c}{a^2\lambda}\left\{1 - \omega e\left(\tau^o\right)\left[1 - G\left(\underline{u}^A\right)\right]\right\}\tau^o < m^c
\end{aligned}
\tag{30}
$$

4.2.3 Summary

Table 1: Summary of Core Results

Assumption	Structural deficit rule	Actual deficit rule
Perfect reputation	τ^o	$\frac{\tau^o}{G(\bar{u})-G(\underline{u})}$
Imperfect reputation	$\frac{\tau^o}{\omega}$	$\frac{\tau^o}{\omega[G(\bar{u}^A)-G(\underline{u}^A)]}$
Costly announcements	I. $\tau_s^A e\left(\tau_s^A\right) = \frac{\tau^o}{\omega}$, if exists	I. $\tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right] = \frac{\tau^o}{\omega}$, if exists
	II. $\arg\max \tau_s^A e\left(\tau_s^A\right)$ otherwise	II. no equilibrium otherwise

In this sub-section we have demonstrated that if higher announced deficit penalties lead to diminished beliefs that penalties will actually be applied, then a principal who initially lacks a reputation may wind up in a credibility trap. There will be no announced penalty that will be believed in equilibrium by the fiscal authority and which will induce an optimal choice of fiscal policy. The possibility of this credibility trap exists for rules written on the actual deficit, but it does not exist for a structural deficit rule, precisely because structural deficit rules require smaller marginal penalties to be effective, and because the actual deficit rule has to include an escape clause. Our core results regarding the structural and actual deficit rules can be summarized in Table 1 below. The case labelled "perfect reputation" covers instances where there are shocks to the economy but where the principal has a perfect reputation. We then provide the equilibrium rules (when they exist) for the case of an imperfect initial reputation, as well as for the case where there is both an imperfect initial reputation, and when announcing higher penalties is 'costly' in the sense that it implies lower probability of enforcement.

5 Conclusion

In this paper we have investigated the effect of fiscal rules under the realistic assumption that fiscal policy outcomes depend on automatic stabilizers and that when rules are first established they may be imperfectly credible. Using a simple model that allows for monetary-fiscal interactions, we have provided formal support for an argument commonly made with regard to the EU Stability Pact; a rule written on the structural deficit is preferable to one written on the actual deficit, provided that estimates of the output gap and of automatic stabilizers are verifiable. Under such a rule, uncertainty in measuring the structural deficit can be dealt with by careful specification of error bands. One of the reasons for preferring a structural

deficit rule is that contracting on the actual deficit will induce a procyclical bias in fiscal policy. In addition to providing formal support for this existing argument, we have also argued that fiscal rules written on the actual deficit may suffer from a "credibility trap". Imperfect initial credibility makes it necessary to announce higher marginal penalties for violations of a rule, but announcing higher marginal penalties may also reduce credibility if the magnitude of such penalties is disproportionately large relative to the violations. This problem is likely to be particularly severe for actual deficit (as opposed to structural deficit) rules, and it will result in an expansionary bias for fiscal policy and a contractionary bias for monetary policy.

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A Proof of Proposition 1.

To solve for the optimal and credible (subgame perfect, or dynamically consistent) delegation parameters, we use backward induction. At stage (iv), policy authorities minimise independently the delegated loss functions, taking delegation as given. Expectations are pinned down as described in the text at (ii). At stage (i) the common principal sharing the social loss uses as instruments the marginal penalty τ and the price level target π^o. Hence, at (iv) authorities solve $\min\limits_{m} L^{MD}(.)$, $\min\limits_{f} L^{FD}(.)$ leading to the first order conditions, respectively:

$$(a + bc)\lambda \left(y - y^F\right) + c\left(\pi - \pi^F\right) + \tau \;=\; 0 \tag{A1}$$

$$b\lambda\left(y - y^M\right) + \pi - \pi^o \;=\; 0$$

We can now solve for the Nash equilibrium y, π as a function of the delegation parameters:

$$y(\tau, \pi^o) \;=\; y^F + \frac{c}{a\lambda}\left(\pi^F - \pi^o\right) + \frac{bc}{a}\left(y^F - y^M\right) - \frac{1}{a\lambda}\tau \tag{A2}$$

$$\pi(\tau, \pi^o) \;=\; \pi^o - \frac{bc}{a}\left(\pi^F - \pi^o\right) - \left(1 + \frac{bc}{a}\right)b\lambda\left(y^F - y^M\right) - \frac{1}{a\lambda}\tau$$

Plugging these values back in the social loss function $L^c(.)$ and minimising with respect to τ, π^o respectively, we obtain the first order conditions:

$$-\frac{1}{a}\left[y(\tau, \pi^o) - y^c\right] + \frac{b}{a}\left[\pi(\tau, \pi^o) - \pi^c\right] \;=\; 0 \tag{A3}$$

$$-\frac{c}{a}\left[y(\tau, \pi^o) - y^c\right] + \frac{a + bc}{a}\left[\pi(\tau, \pi^o) - \pi^c\right] \;=\; 0$$

Substituting $\pi(\tau, \pi^o), y(\tau, \pi^o)$ and solving for the delegation parameters, after straightforward algebra we obtain these τ^o, π^o as in Proposition 1.

The above solves for the subgame perfect (hence dynamically consistent or credible) delegation parameters. To show this policy design mechanism implements the optimal policy mix, plug these values back in equation (A2), which delivers: $y(\tau^o, \pi^o) = y^c, \pi(\tau^o, \pi^o) = \pi^c$. Hence, the deficit contract and price level target are both optimal and time consistent. ∎

B Proof of Proposition 2

Consider the problem of the fiscal authority $\min\limits_{s} E_{\omega, e(.)}\left[L^F\right] = L^F + \omega\tau_s^A e\left(\tau_s^A\right)\left[s - \frac{1}{a}y^c\right] + (1 - \omega)\,0$ and the usual problem of the monetary authority under the price level target, which leads to the first order conditions (where $\int_u x$ is shorthand notation for expected value of x with

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respect to the distribution of u, distribution which is common knowledge to policy authorities and private sector):

$$(a + bc)\lambda \left(\int_u y - y^F \right) + c \left(\int_u \pi - \pi^F \right) + \omega \tau_s^A e \left(\tau_s^A \right) = 0 \tag{B1}$$

$$b\lambda \left(\int_u y - y^M \right) + \pi - \pi^o = 0$$

and rewrite as

$$(a + bc)\lambda \left(\int_u y - y^c \right) + c \left(\int_u \pi - \pi^c \right) = \left[\tau^o - \omega \tau_s^A e \left(\tau_s^A \right) \right] \tag{B2}$$

$$b\lambda \left(\int_u y - y^c \right) + \int_u \pi - \pi^c = 0$$

Solving for $\int_u y \left(\tau_s^A e \left(\tau_s^A \right) \right), \int_u \pi \left(\tau_s^A e \left(\tau_s^A \right) \right)$, plugging back in $L^c (.)$ after algebraic manipulation we get the objective of the principal taking into account the reaction of the two policy authorities:

$$L^c \left(\tau_s^A \right) = \frac{1}{2} \left[\frac{1}{a^2 \lambda} \left(\tau^o - \omega \tau_s^A e \left(\tau_s^A \right) \right)^2 + \frac{b^2}{a^2} \left(\omega \tau_s^A e \left(\tau_s^A \right) - \tau^o \right)^2 \right] \tag{B3}$$

and solving $\min_{\tau_s^A} EL^c \left[y \left(\tau_s^A e \left(\tau_s^A \right) \right), \pi \left(\tau_s^A e \left(\tau_s^A \right) \right) \right]$ after straightforward algebra we get the first order necessary condition for the optimal announcement τ_s^A:

$$\omega \left[\omega \tau_s^A e \left(\tau_s^A \right) - \tau^o \right] \left[\tau_s^A e \left(\tau_s^A \right) \right]' \left[\frac{1 + b^2 \lambda}{a^2 \lambda} \right] = 0 \tag{B4}$$

The stationary points of this are given by (since the third term in brackets is always bigger than zero.):

$$\tau_s^A e \left(\tau_s^A \right) = \frac{\tau^o}{\omega} \text{ or } \left[\tau_s^A e \left(\tau_s^A \right) \right]' = e \left(\tau_s^A \right) + \tau_s^A e' \left(\tau_s^A \right) = 0 \tag{B5}$$

For sufficiency, we need to check second order conditions, which are given by (dropping irrelevant multiplicative constants):

$$\omega \left\{ \left[\tau_s^A e \left(\tau_s^A \right) \right]' \right\}^2 + \left[\omega \tau_s^A e \left(\tau_s^A \right) - \tau^o \right] \left[\tau_s^A e \left(\tau_s^A \right) \right]'' > 0 \tag{B6}$$

From B6 we see that both FOCs cannot hold at the same time as both terms in ?? would be zero. So we can distinguish: Case I. It is obvious by direct inspection of $L^c \left(\tau_s^A \right)$ that $\tau_s^A e \left(\tau_s^A \right) = \frac{\tau^o}{\omega}$ implicitly defines an equilibrium whenever it has a solution (note this is a nonlinear equation). The bad news is that even in this case there is more than one equilibrium (at least two) on the $[0, \bar{\tau}]$ interval. Under plausible assumption on the form of $e(.)$ and its

derivatives, there are two equilibria such that $\tau_s^A e\left(\tau_s^A\right) = \frac{\tau^o}{\omega}$ - an intuitive interpretation of the conditions is given in text; Case II: However, it might be the case that there is no such equilibrium, i.e. for $\tau_s^A e\left(\tau_s^A\right) < \frac{\tau^o}{\omega}$ for any τ_s^A, which is equivalent (under continuity) to $\max_{\tau_s^A}\tau_s^A e\left(\tau_s^A\right) < \frac{\tau^o}{\omega}$. Then, the equilibrium is given by the solution to $\left[\tau_s^A e\left(\tau_s^A\right)\right]' = e\left(\tau_s^A\right) + \tau_s^A e'\left(\tau_s^A\right) = 0$, which defines a unique equilibrium for $\left[\tau_s^A e\left(\tau_s^A\right)\right]'' < 0$, as can be seen from the second order condition.

C Proof of Proposition 3

The proof is similar to that of Proposition 2, the only difference being that now we have an additional control variable for the principal, namely the announcement of the threshold for the shock $\underline{u}$. As this mirrors the proof of Proposition 2 we shall skip some obvious steps to save space. Given an announcement for the marginal penalty τ^A and the escape clause $\underline{u}^A$, one first finds the Nash equilibrium policy instrument rules followed by the monetary and fiscal authority, and hence the equilibrium outcomes as functions of the marginal penalty and the threshold. The obtained values are then substituted into the loss function of the principal to obtain:

$$L^c\left(\tau_f^A, \underline{u}^A\right) = \frac{1}{2}\left[\frac{1}{a^2\lambda}\left(\tau^o - \omega\tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right]\right)^2 + \frac{b^2}{a^2}\left(\omega\tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right] - \tau^o\right)^2\right]$$
(C1)

Solving $\min\limits_{\tau_f^A, \underline{u}^A} \int L^c\left(\tau_f^A, \underline{u}^A\right) dG\left(u\right)$ leads to the necessary first order conditions, for each of the announcements respectively:

$$\frac{\partial EL^c}{\partial \tau_f^A} = \omega\left[1 - G\left(\underline{u}^A\right)\right]\left\{\omega\tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right] - \tau^o\right\}\left[\tau_f^A e\left(\tau_f^A\right)\right]'\left[\frac{1 + b^2\lambda}{a^2\lambda}\right] = 0 \text{(C2)}$$

$$\frac{\partial EL^c}{\partial \underline{u}^A} = \omega\left[-G'\left(\underline{u}^A\right)\right]\left[\tau_f^A e\left(\tau_f^A\right)\right]\left\{\omega\tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right] - \tau^o\right\}\left[\frac{1 + b^2\lambda}{a^2\lambda}\right] = 0$$

One stationary point would be $\tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right] = \frac{\tau^o}{\omega}$. This describes a schedule in the $\tau_f^A, \underline{u}^A$ space and has solutions if f $\max\limits_{\tau_f^A}\tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right] > \frac{\tau^o}{\omega}$ (case I in proposition 3). One immediately sees that this is indeed a minimum by direct inspection of the loss function (C1) evaluated at any such point giving $L^c\left(\tau_f^A, \underline{u}^A\right) = 0$ and noting that the function is strictly greater than zero otherwise. Hence, we now look for equilibria in Case II, where $\max\limits_{\tau_f^A}\tau_f^A e\left(\tau_f^A\right)\left[1 - G\left(\underline{u}^A\right)\right] < \frac{\tau^o}{\omega}$. The next thing to observe is that we can rule out cases where

$G\left(\underline{u}^{A}\right)=1$ (which means no rule on the whole support of shocks) and where $G'\left(\underline{u}^{A}\right)=0$ (i.e. $\underline{u}^{A}$ is an improbable event and then we end up with a rule for the whole range of shocks). The remaining stationary points are $\left[\tau_{f}^{A}e\left(\tau_{f}^{A}\right)\right]'=0$ and either $\tau_{f}^{A}=0$ or $e\left(\tau_{f}^{A}\right)=0$. $\tau_{f}^{A}=0$ cannot be satisfied as long as $\left[\tau_{f}^{A}e\left(\tau_{f}^{A}\right)\right]'=0$, so we are left with $e\left(\tau_{f}^{A}\right)=0$, $\left[\tau_{f}^{A}e\left(\tau_{f}^{A}\right)\right]'=0$ which is satisfied for any $\tau_{f}^{A}>\bar{\tau}$. But this does not satisfy second order conditions for a minimum: first we look at $\frac{\partial^{2}EL^{c}}{\partial\tau_{f}^{A2}}$ and check whether it is >0 which together with $\frac{\partial^{2}EL^{c}}{\partial\tau_{f}^{A2}}\frac{\partial^{2}EL^{c}}{\partial\underline{u}^{A2}}-$ $\left(\frac{\partial^{2}EL^{c}}{\partial\tau_{f}^{A}\partial\underline{u}^{A}}\right)^{2}>0$ would be the sufficient conditions for a minimum:

$$\frac{\partial^{2}EL^{c}}{\partial\tau_{f}^{A2}}=\omega^{2}\left[1-G\left(\underline{u}^{A}\right)\right]^{2}\left\{\left[\tau_{f}^{A}e\left(\tau_{f}^{A}\right)\right]'\right\}^{2}+\left\{\omega\tau_{f}^{A}e\left(\tau_{f}^{A}\right)\left[1-G\left(\underline{u}^{A}\right)\right]-\tau^{o}\right\}\left[\tau_{f}^{A}e\left(\tau_{f}^{A}\right)\right]''\overset{\forall\tau_{f}^{A}>\bar{\tau}}{=}0$$

(31)

We can see again why we impose $G\left(\underline{u}^{A}\right)\neq1$ ($\frac{\partial^{2}EL^{c}}{\partial\tau_{f}^{A2}}$ would be zero), and $e\left(\tau_{f}^{A}\right)=0$ is not a minimum (hence, we do not have to check for the determinant of the Hessian being greater than zero). In case II there is no equilibrium announcement.