Least Trimmed Squares: Cointegration and outliers SUPPLEMENT Further Simulations

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S Further Simulations

We consider the following model

$$\Delta y_t = \omega \Delta z_t + \alpha(y_{t-1} - \kappa z_{t-1} - \nu) + \sigma_\varepsilon \varepsilon_t, \tag{S.1}$$

$$\Delta z_t = \sigma_\eta \eta_t, \tag{S.2}$$

where ε_t and η_t are independent of each other. We set $\omega = 0.5$, $\alpha = -1$ or $\alpha = -0.2$, $\kappa = 1$, $\nu = 1$ and $\sigma_\varepsilon = \sigma_\eta = 1$ while $z_0 = 0$ and $y_0 = \nu_c$.

We study the performance of t-statistics for $\omega, \alpha, \kappa, \nu_c$ when computed using OLS, LTS and SLTS inference, as in Berenguer-Rico and Nielsen (2024). To be more specific, let $\psi = -\alpha\kappa$ and $\mu = -\alpha\nu$ and reformulate model (S.1) as

$$\Delta y_t = \omega \Delta w_t + \alpha y_{t-1} + \psi z_{t-1} + \mu + \sigma_\varepsilon \varepsilon_t, \tag{S.3}$$

and estimate using both the OLS and LTS estimators. For the LTS estimator we apply inference using two different approaches. On the one hand, we use inference as established in this paper when using the LTS model and the correct value of h. We refer to this as LTS. On the other hand, inference is conducted using the same LTS estimator, where the same true h is chosen, but assuming an uncontaminated normal model for inference. We refer to this as SLTS, for standard LTS, as this inference approach is customary in the robust statistics literature.

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Tests for ω and α are direct. Hence, to test $\mathsf{H}_0 : \omega = \omega_0$ and $\mathsf{H}_0 : \alpha = \alpha_0$, for some hypothesised values ω_0, α_0, we use

$$t_{\omega,s} = \frac{\hat{\omega}_s - \omega_0}{\mathsf{s.e.}(\hat{\omega}_s)} \quad \text{and} \quad t_{\alpha,s} = \frac{\hat{\alpha}_s - \alpha_0}{\mathsf{s.e.}(\hat{\alpha}_s)},$$

respectively, where $s \in \{OLS, LTS, SLTS\}$ and the standard errors $\mathsf{s.e.}(\hat{\omega}_s)$ and $\mathsf{s.e.}(\hat{\alpha}_s)$ vary with s as follows. Let $R_\omega = (1,0,0,0)'$ and $R_\alpha = (0,1,0,0)'$. Then,

$$\mathsf{s.e.}(\hat{\omega}_s) = \sqrt{s_s^2(\hat{R}_\kappa)' M_s(\hat{R}_\kappa)}, \quad \mathsf{s.e.}(\hat{\alpha}_s) = \sqrt{s_s^2(\hat{R}_\alpha)' M_s(\hat{R}_\alpha)},$$

where $s_{OLS}^2 = (T-4)^{-1}\sum_{i=1}^T (y_t - x_t'\theta)^2$, $s_{LTS}^2 = (T-4)^{-1}\sum_{t\in\hat{\zeta}} (y_t - x_t'\theta)^2$ and $s_{SLTS}^2 = s_{LTS}^2/\varsigma_{\lambda,\Phi}^4$, while $M_{OLS} = (\sum_{t=1}^T x_t x_t')^{-1}$ and $M_{LTS} = M_{SLTS} = (\sum_{t\in\hat{\zeta}} x_t x_t')^{-1}$.

Tests for κ and ν are indirect. Hence, to test $\mathsf{H}_0 : \kappa = \kappa_0$ and $\mathsf{H}_0 : \nu = \nu_0$, for hypothesised values κ_0, ν_0, we use

$$t_{\kappa,s} = \frac{\hat{\kappa}_s - \kappa_0}{\mathsf{s.e.}(\hat{\kappa}_s)} = \frac{(-\hat{\psi}_s/\hat{\alpha}_s) - \kappa_0}{\mathsf{s.e.}(\hat{\psi}_s/\hat{\alpha}_s)} \qquad \textit{and} \qquad t_{\nu,s} = \frac{\hat{\nu}_s - \nu_0}{\mathsf{s.e.}(\hat{\nu}_s)} = \frac{(-\hat{\mu}_s/\hat{\alpha}_s) - \nu_0}{\mathsf{s.e.}(\hat{\mu}_s/\hat{\alpha}_s)},$$

where the standard errors $\mathsf{s.e.}(\hat{\kappa}_s) = \mathsf{s.e.}(\hat{\psi}_s/\hat{\alpha}_s)$ and $\mathsf{s.e.}(\hat{\nu}_s) = \mathsf{s.e.}(\hat{\mu}_s/\hat{\alpha}_s)$ vary with s and can be obtained using the delta method. To that end, let

$$\phi_1(\theta) = -\frac{\psi}{\alpha} \qquad \text{with} \qquad D_\kappa = \frac{\partial \phi_1(\theta)}{\partial \theta} = \begin{pmatrix} \partial\phi_1/\partial\omega \\ \partial\phi_1/\partial\alpha \\ \partial\phi_1/\partial\psi \\ \partial\phi_1/\partial\mu \end{pmatrix} = \begin{pmatrix} 0 \\ \psi/\alpha^2 \\ -1/\alpha \\ 0 \end{pmatrix},$$

and

$$\phi_2(\theta) = -\frac{\mu}{\alpha} \qquad \text{with} \qquad D_\nu = \frac{\partial \phi_2(\theta)}{\partial \theta} = \begin{pmatrix} \partial\phi_2/\partial\omega \\ \partial\phi_2/\partial\alpha \\ \partial\phi_2/\partial\psi \\ \partial\phi_2/\partial\mu \end{pmatrix} = \begin{pmatrix} 0 \\ \mu/\alpha^2 \\ 0 \\ -1/\alpha \end{pmatrix}.$$

For $s \in \{OLS, LTS, SLTS\}$ define $\hat{D}_\kappa^s$ and $\hat{D}_\nu^s$ as the vectors D_κ and D_ν evaluated at the s estimator where recall $\hat{\theta}_{LTS} = \hat{\theta}_{SLTS}$. Then,

$$\mathsf{s.e.}(\hat{\kappa}_s) = \mathsf{s.e.}(\hat{\psi}_s/\hat{\alpha}_s) = \sqrt{s_s^2(\hat{D}_\kappa^s)' M_s(\hat{D}_\kappa^s)}, \quad \mathsf{s.e.}(\hat{\nu}_s) = \mathsf{s.e.}(\hat{\mu}_s/\hat{\alpha}_s) = \sqrt{s_s^2(\hat{D}_\nu^s)' M_s(\hat{D}_\nu^s)}.$$

We study 18 different data generating processes (DGPs). In all DGPs there is contamination of LTS type in both ε_t and η_t. In the different simulation configurations, we vary the sample size as well as the number and location of outliers in the sample as follows. Let ζ_h be the set of good observations and its complement ζ_h^c the set of outliers. For $t \in \zeta_h$, we set $\eta_t \sim i.i.d.\mathsf{N}(0,1)$ and $\varepsilon_t \sim i.i.d.\mathsf{N}(0,1)$. For $s \notin \zeta_h$, we set $\eta_s = \sqrt{2\log h} + \xi_{\eta t} + c_\eta$ and $\varepsilon_s = \sqrt{2\log h} + \xi_{\varepsilon t} + c_\varepsilon$ with $c_\eta = 0$ and $c_\varepsilon = 10$. Then, w_t and y_t follow the model in equations (S.1) and (S.2).

In DGPs 1-6, outliers in both ε_t and η_t are located in the middle of the sample so that

$$\zeta_h^c = \{\lceil h/2 \rceil + 1, \lceil h/2 \rceil + 2, \ldots, \lceil h/2 \rceil + (T-h)\},$$

where $\lceil \cdot \rceil$ denotes the ceiling function.

In DGPs 7-12, the $T-h$ outliers occur in $G=5$ episodes and in both ε_t and η_t. In each episode there are $\lfloor (T-h)/G \rfloor$ outliers. Outlier episodes are equally spaced by $\lfloor h/G \rfloor$ good observations. Specifically, the system starts with $\lfloor h/G \rfloor$ good observations, after which there is an episode with $\lfloor (T-h)/G \rfloor$ outliers. This is followed by another $\lfloor h/G \rfloor$ good observations, after which another episode with $\lfloor (T-h)/G \rfloor$ outliers follows. This repeats for $G=5$ episodes in the sample.

In DGPs 13-18, there are $T-h$ outliers in both ε_t and η_t. These occur individually through the sample. Specifically, if $T/(T-h)$ is an integer, the system starts with $\{T/(T-h)\}-1$ good observations, after which there is one single outlier at $T/(T-h)$. This is followed by $\{T/(T-h)\}-1$ good observations, after which there is another single outlier. This repeats for the $T-h$ outliers in the sample.

If $T/(T-h)$ is not an integer, then the system starts with $\lceil T/(T-h) \rceil - 2$ good observations, after which there is one single outlier at $\lceil T/(T-h) \rceil - 1$. This is followed by $\lceil T/(T-h) \rceil - 2$ good observations, after which there is another single outlier. This repeats for the $T-h$ outliers in the sample.

Number of repetitions: 10^4.

Table 1 describes the number of outliers, $T-h$, in each DGP.

Table 1: DGPs: Data Generating Processes.

Middle	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
$T-h$	$\lceil (1/2)\sqrt{T} \rceil$	$\lceil \sqrt{T} \rceil$	$\lceil 2\sqrt{T} \rceil$	$\lceil 0.04T \rceil$	$\lceil 0.08T \rceil$	$\lceil 0.16T \rceil$
5 episodes	DGP7	DGP8	DGP9	DGP10	DGP11	DGP12
$T-h$	$\lceil (1/2)\sqrt{T} \rceil$	$\lceil \sqrt{T} \rceil$	$\lceil 2\sqrt{T} \rceil$	$\lceil 0.05T \rceil$	$\lceil 0.10T \rceil$	$\lceil 0.20T \rceil$
Individual	DGP13	DGP14	DGP15	DGP16	DGP17	DGP18
$T-h$	$\lceil (1/2)\sqrt{T} \rceil$	$\lceil \sqrt{T} \rceil$	$\lceil 2\sqrt{T} \rceil$	$\lceil 0.04T \rceil$	$\lceil 0.10T \rceil$	$\lceil 0.20T \rceil$

In DGPs 1-6, the outliers are consecutive and located in the middle of the sample.
In DGPs 7-12, the outliers occur in 5 distinct episodes. No $T=25$ in DGPs 7, 10, 11.
In DGPs 13-18, the outliers occur individually. No $T=25$ in DGP 15.

Table 2: Simulated rejection frequencies for nominal 5% tests on ω.

method	n	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
OLS	25	0.9840	0.9870	0.9906	0.8453	0.9732	0.9883
	100	0.9996	0.9999	0.9999	0.9995	0.9998	0.9999
	400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
LTS	25	0.0773	0.2722	0.7241	0.0627	0.0644	0.1393
	100	0.0520	0.0563	0.6851	0.0525	0.0540	0.4092
	400	0.0518	0.0523	0.0546	0.0505	0.0537	0.7113
	1600	0.0470	0.0529	0.0451	0.0498	0.0502	0.9076
	6400	0.0507	0.0507	0.0507	0.0502	0.0514	0.2977
SLTS	25	0.1615	0.4308	0.8692	0.0939	0.1270	0.2566
	100	0.0931	0.1261	0.7922	0.0844	0.1129	0.5059
	400	0.0708	0.0893	0.1236	0.0832	0.1139	0.7524
	1600	0.0581	0.0749	0.0852	0.0816	0.1140	0.9276
	6400	0.0581	0.0629	0.0722	0.0796	0.1068	0.3802

In DGPs 1-6, the $T - h$ outliers are located in the middle of the sample. In this table $\alpha = -1$.

Table 3: Simulated rejection frequencies for nominal 5% tests on ω.

method	n	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
OLS	25	0.9979	0.9997	0.9989	0.8567	0.9931	0.9993
	100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
LTS	25	0.0674	0.0710	0.7839	0.0630	0.0676	0.0664
	100	0.0526	0.0503	0.2796	0.0534	0.0529	0.0836
	400	0.0520	0.0523	0.3616	0.0501	0.3277	0.4375
	1600	0.0471	0.0942	0.6515	0.6481	0.6609	0.6734
	6400	0.0509	0.9920	0.1581	0.9924	0.9873	0.9661
SLTS	25	0.1544	0.2127	0.8863	0.0969	0.1264	0.1864
	100	0.0916	0.1255	0.4633	0.0854	0.1107	0.2003
	400	0.0718	0.0867	0.4808	0.0823	0.4354	0.5813
	1600	0.0583	0.1165	0.7258	0.7060	0.7442	0.7725
	6400	0.0574	0.9935	0.1767	0.9957	0.9929	0.9807

In DGPs 1-6, the $T - h$ outliers are located in the middle of the sample. In this table $\alpha = -0.2$.

Table 4: Simulated rejection frequencies for nominal 5% tests on α.

method	n	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
OLS	25	0.5970	0.8378	0.8583	0.0172	0.2175	0.7676
	100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
LTS	25	0.0946	0.5200	0.9986	0.0597	0.0669	0.2291
	100	0.0519	0.0598	0.9221	0.0471	0.0542	0.4995
	400	0.0484	0.0529	0.0506	0.0516	0.0508	0.7168
	1600	0.0505	0.0481	0.0507	0.0496	0.0499	0.9099
	6400	0.0530	0.0490	0.0520	0.0525	0.0521	0.3024
SLTS	25	0.1810	0.6068	0.9993	0.0926	0.1260	0.3311
	100	0.0891	0.1371	0.9421	0.0807	0.1133	0.5637
	400	0.0694	0.0938	0.1224	0.0837	0.1099	0.7611
	1600	0.0626	0.0676	0.0890	0.0796	0.1112	0.9285
	6400	0.0586	0.0586	0.0745	0.0856	0.1055	0.3872

In DGPs 1-6, the $T - h$ outliers are located in the middle of the sample. In this table $\alpha = -1$

Table 5: Simulated rejection frequencies for nominal 5% tests on α.

method	n	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
OLS	25	0.0320	0.1726	0.4991	0.1055	0.0113	0.0962
	100	0.9984	1.0000	1.0000	0.9666	1.0000	1.0000
	400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
LTS	25	0.0752	0.0748	0.7941	0.0901	0.0752	0.0764
	100	0.0555	0.0560	0.8102	0.0565	0.0606	0.1262
	400	0.0532	0.0549	1.0000	0.0516	0.9809	1.0000
	1600	0.0498	0.1190	1.0000	0.9896	1.0000	1.0000
	6400	0.0536	1.0000	0.1624	1.0000	1.0000	1.0000
SLTS	25	0.1701	0.2218	0.8922	0.1265	0.1393	0.1997
	100	0.0970	0.1253	0.8430	0.0884	0.1191	0.2344
	400	0.0717	0.0959	1.0000	0.0805	0.9820	1.0000
	1600	0.0614	0.1386	1.0000	0.9903	1.0000	1.0000
	6400	0.0601	1.0000	0.1822	1.0000	1.0000	1.0000

In DGPs 1-6, the $T - h$ outliers are located in the middle of the sample. In this table $\alpha = -0.2$.

Table 6: Simulated rejection frequencies for nominal 5% tests on κ.

method	n	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
OLS	25	0.0343	0.0340	0.0553	0.0593	0.0395	0.0310
	100	0.0134	0.0053	0.0055	0.0161	0.0071	0.0053
	400	0.0033	0.0000	0.0000	0.0004	0.0000	0.0000
	1600	0.0059	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	0.0274	0.0006	0.0000	0.0000	0.0000	0.0000
LTS	25	0.0705	0.0760	0.3619	0.0673	0.0706	0.0693
	100	0.0568	0.0531	0.0762	0.0536	0.0548	0.0473
	400	0.0558	0.0544	0.0548	0.0583	0.0530	0.0378
	1600	0.0493	0.0529	0.0486	0.0495	0.0518	0.0306
	6400	0.0453	0.0518	0.0508	0.0516	0.0489	0.0565
SLTS	25	0.1564	0.2117	0.6537	0.1007	0.1283	0.1808
	100	0.0983	0.1251	0.2255	0.0881	0.1160	0.1459
	400	0.0770	0.0957	0.1245	0.0899	0.1098	0.1307
	1600	0.0605	0.0736	0.0890	0.0790	0.1099	0.1258
	6400	0.0537	0.0633	0.0728	0.0837	0.1078	0.1763

In DGPs 1-6, the $T - h$ outliers are located in the middle of the sample. In this table $\alpha = -1$

Table 7: Simulated rejection frequencies for nominal 5% tests on κ.

method	n	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
OLS	25	0.0524	0.0152	0.0050	0.1694	0.1237	0.0280
	100	0.0129	0.0599	0.3134	0.0112	0.0317	0.1999
	400	0.0131	0.0275	0.1128	0.0184	0.0695	0.2912
	1600	0.0000	0.0027	0.3776	0.1579	0.7799	0.9716
	6400	0.0000	0.0039	0.0494	0.0847	0.1432	0.2309
LTS	25	0.0890	0.0820	0.6648	0.0933	0.0906	0.0903
	100	0.0626	0.0570	0.3636	0.0586	0.0563	0.0910
	400	0.0575	0.0565	0.1321	0.0561	0.0760	0.3192
	1600	0.0520	0.0499	0.0293	0.0154	0.0884	0.2937
	6400	0.0472	0.0001	0.0472	0.0178	0.0850	0.2802
SLTS	25	0.1670	0.2175	0.8432	0.1241	0.1474	0.1962
	100	0.1019	0.1245	0.5925	0.0936	0.1155	0.2167
	400	0.0779	0.0954	0.2422	0.090	0.1518	0.4934
	1600	0.0609	0.0688	0.0563	0.0284	0.1644	0.4601
	6400	0.0545	0.0002	0.0656	0.0340	0.1573	0.4406

In DGPs 1-6, the $T - h$ outliers are located in the middle of the sample. In this table $\alpha = -0.2$.

Table 8: Simulated rejection frequencies for nominal 5% tests on ν.

method	n	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
OLS	25	0.0367	0.0729	0.1409	0.0425	0.0225	0.0553
	100	0.0089	0.0273	0.0484	0.0092	0.0192	0.0401
	400	0.0140	0.0116	0.0079	0.0123	0.0072	0.0071
	1600	0.1946	0.3112	0.4893	0.4168	0.6314	0.4469
	6400	0.6169	0.9366	1.0000	1.0000	1.0000	1.0000
LTS	25	0.0803	0.1064	0.6611	0.0740	0.0793	0.0790
	100	0.0606	0.0603	0.1413	0.0600	0.0576	0.0633
	400	0.0562	0.0553	0.0582	0.0530	0.0561	0.0784
	1600	0.0475	0.0509	0.0538	0.0525	0.0532	0.1752
	6400	0.0501	0.0515	0.0511	0.0527	0.0548	0.1915
SLTS	25	0.1610	0.2158	0.7347	0.1032	0.1324	0.1743
	100	0.0942	0.1270	0.7922	0.0889	0.1157	0.1580
	400	0.0762	0.0917	0.2495	0.0839	0.1077	0.1786
	1600	0.0577	0.0702	0.0924	0.0822	0.1071	0.3008
	6400	0.0557	0.0633	0.0718	0.0816	0.1088	0.3013

In DGPs 1-6, the $T - h$ outliers are located in the middle of the sample. In this table $\alpha = -1$

Table 9: Simulated rejection frequencies for nominal 5% tests on ν.

method	n	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
OLS	25	0.0472	0.0284	0.0297	0.1480	0.0857	0.0316
	100	0.0041	0.0593	0.2591	0.0048	0.0261	0.1808
	400	0.0020	0.0068	0.0641	0.0022	0.0312	0.1955
	1600	0.0000	0.0000	0.0349	0.0081	0.1611	0.3689
	6400	0.0000	0.0134	0.1482	0.2111	0.2934	0.3780
LTS	25	0.0866	0.0868	0.6259	0.1043	0.0939	0.0811
	100	0.0617	0.0614	0.0834	0.0601	0.0602	0.0630
	400	0.0565	0.0543	0.0769	0.0550	0.0329	0.2644
	1600	0.0485	0.0485	0.0372	0.0206	0.1137	0.3656
	6400	0.0506	0.0023	0.0504	0.0807	0.2016	0.4238
SLTS	25	0.1669	0.2173	0.7648	0.1311	0.1443	0.1860
	100	0.0953	0.1279	0.2157	0.0923	0.1152	0.1777
	400	0.0776	0.0930	0.1499	0.0851	0.0752	0.395
	1600	0.0599	0.0652	0.0671	0.0369	0.1822	0.4792
	6400	0.0562	0.0034	0.0703	0.1178	0.2828	0.5297

In DGPs 1-6, the $T - h$ outliers are located in the middle of the sample. In this table $\alpha = -0.2$.

Table 10: Simulated rejection frequencies for nominal 5% tests on ω.

method	n	DGP7	DGP8	DGP9	DGP10	DGP11	DGP12
OLS	25		1.0000	1.0000			1.0000
	100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
LTS	25		0.0661	0.6911			0.0661
	100	0.0546	0.0531	0.2729	0.0546	0.0531	0.2729
	400	0.0527	0.0541	0.0493	0.0541	0.0493	0.9822
	1600	0.0482	0.0519	0.0496	0.0496	0.0529	1.0000
	6400	0.0472	0.0495	0.0478	0.0527	0.0493	1.0000
SLTS	25		0.2101	0.8229			0.2101
	100	0.0989	0.1269	0.4017	0.0989	0.1269	0.4017
	400	0.0737	0.0933	0.1184	0.0933	0.1184	0.9900
	1600	0.0577	0.0743	0.0857	0.0857	0.1283	1.0000
	6400	0.0533	0.0607	0.0680	0.0933	0.1172	1.0000

In DGPs 7-12, the $T - h$ outliers are equally distributed in G outlier episodes. The episodes are equally spaced through the sample. In this table $\alpha = -1$

Table 11: Simulated rejection frequencies for nominal 5% tests on ω.

method	n	DGP7	DGP8	DGP9	DGP10	DGP11	DGP12
OLS	25		1.0000	1.0000			1.0000
	100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
LTS	25		0.0697	0.7343			0.0697
	100	0.0561	0.0527	0.0513	0.0561	0.0527	0.0513
	400	0.0515	0.0555	0.0510	0.0555	0.0510	0.2549
	1600	0.0463	0.0512	0.0503	0.0503	0.8208	0.7750
	6400	0.0472	0.0485	0.0479	0.9957	0.9913	0.9666
SLTS	25		0.2133	0.8332			0.2133
	100	0.0978	0.1266	0.2019	0.0978	0.1266	0.2019
	400	0.0718	0.0931	0.1181	0.0931	0.1181	0.3919
	1600	0.0577	0.0723	0.0856	0.0856	0.8850	0.8667
	6400	0.0528	0.0609	0.0675	0.9983	0.9957	0.9817

In DGPs 7-12, the $T - h$ outliers are equally distributed in G outlier episodes. The episodes are equally spaced through the sample. In this table $\alpha = -0.2$

Table 12: Simulated rejection frequencies for nominal 5% tests on α.

method	n	DGP7	DGP8	DGP9	DGP10	DGP11	DGP12
OLS	25		0.0884	0.0385			0.0884
	100	0.0219	0.9409	0.9999	0.0219	0.9409	0.9999
	400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
LTS	25		0.0722	0.5261			0.0722
	100	0.0525	0.0527	0.3365	0.0525	0.0527	0.3365
	400	0.0539	0.0518	0.0518	0.0518	0.0518	0.9922
	1600	0.0484	0.0516	0.0514	0.0514	0.0475	1.0000
	6400	0.0518	0.0527	0.0494	0.0499	0.0502	1.0000
SLTS	25		0.2186	0.7522			0.2186
	100	0.0951	0.1237	0.4469	0.0951	0.1237	0.4469
	400	0.0760	0.0878	0.1248	0.0878	0.1248	0.9947
	1600	0.0589	0.0719	0.0848	0.0848	0.1164	1.0000
	6400	0.0582	0.0631	0.0697	0.0882	0.1258	1.0000

In DGPs 7-12, the $T - h$ outliers are equally distributed in G outlier episodes. The episodes are equally spaced through the sample. In this table $\alpha = -1$

Table 13: Simulated rejection frequencies for nominal 5% tests on α.

method	n	DGP7	DGP8	DGP9	DGP10	DGP11	DGP12
OLS	25		0.4484	0.1421			0.4484
	100	0.1812	0.0056	0.1792	0.1812	0.0056	0.1792
	400	0.8642	1.0000	1.0000	1.0000	1.0000	1.0000
	1600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
LTS	25		0.0827	0.4726			0.0827
	100	0.0584	0.0536	0.0542	0.0584	0.0536	0.0542
	400	0.0540	0.0521	0.0527	0.0521	0.0527	0.3168
	1600	0.0494	0.0494	0.0468	0.0468	1.0000	1.0000
	6400	0.0518	0.0545	0.0528	1.0000	1.0000	1.0000
SLTS	25		0.2339	0.7112			0.2339
	100	0.0948	0.1295	0.1963	0.0948	0.1295	0.1963
	400	0.0761	0.0885	0.1266	0.0885	0.1266	0.4231
	1600	0.0622	0.0722	0.0888	0.0888	1.0000	1.0000
	6400	0.0582	0.0660	0.0729	1.0000	1.0000	1.0000

In DGPs 7-12, the $T - h$ outliers are equally distributed in G outlier episodes. The episodes are equally spaced through the sample. In this table $\alpha = -0.2$

Table 14: Simulated rejection frequencies for nominal 5% tests on κ.

method	n	DGP7	DGP8	DGP9	DGP10	DGP11	DGP12
OLS	25		0.0560	0.0474			0.0560
	100	0.0227	0.0113	0.0091	0.0227	0.0113	0.0091
	400	0.0021	0.0001	0.0001	0.0001	0.0001	0.0001
	1600	0.0051	0.0000	0.0000	0.0000	0.0000	0.0000
	6400	0.0550	0.0392	0.0101	0.0008	0.0000	0.0000
LTS	25		0.0698	0.3466			0.0698
	100	0.0585	0.0577	0.0450	0.0585	0.0577	0.0450
	400	0.0508	0.0516	0.0545	0.0516	0.0545	0.0315
	1600	0.0529	0.0488	0.0512	0.0512	0.0479	0.0520
	6400	0.0469	0.0489	0.0487	0.0489	0.0504	0.1392
SLTS	25		0.2149	0.6136			0.2149
	100	0.0949	0.1299	0.1721	0.0949	0.1299	0.1721
	400	0.0697	0.0917	0.1285	0.0917	0.1285	0.1475
	1600	0.0672	0.0682	0.0867	0.0867	0.1172	0.1917
	6400	0.0530	0.0599	0.0697	0.0858	0.1210	0.3369

In DGPs 7-12, the $T - h$ outliers are equally distributed in G outlier episodes. The episodes are equally spaced through the sample. In this table $\alpha = -1$

Table 15: Simulated rejection frequencies for nominal 5% tests on κ.

method	n	DGP7	DGP8	DGP9	DGP10	DGP11	DGP12
OLS	25		0.4879	0.3122			0.4879
	100	0.0880	0.0495	0.0299	0.0880	0.0495	0.0299
	400	0.0037	0.0003	0.0003	0.0003	0.0003	0.0020
	1600	0.0011	0.0003	0.0000	0.0000	0.0000	0.0000
	6400	0.0016	0.0000	0.0000	0.0000	0.0000	0.0000
LTS	25		0.1083	0.3649			0.1083
	100	0.0674	0.0630	0.0556	0.0674	0.0630	0.0556
	400	0.0539	0.0533	0.0541	0.0533	0.0541	0.0666
	1600	0.0545	0.0488	0.0515	0.0515	0.0401	0.2186
	6400	0.0481	0.0491	0.0485	0.0179	0.0800	0.2172
SLTS	25		0.2175	0.6354			0.2175
	100	0.1081	0.1340	0.2026	0.1081	0.1340	0.2026
	400	0.0734	0.0924	0.1259	0.0924	0.1259	0.2370
	1600	0.0679	0.0696	0.0866	0.0866	0.1090	0.4203
	6400	0.0540	0.0600	0.0702	0.0378	0.1643	0.4178

In DGPs 7-12, the $T - h$ outliers are equally distributed in G outlier episodes. The episodes are equally spaced through the sample. In this table $\alpha = -0.2$

Table 16: Simulated rejection frequencies for nominal 5% tests on ν.

method	n	DGP7	DGP8	DGP9	DGP10	DGP11	DGP12
OLS	25		0.1263	0.1150			0.1263
	100	0.1293	0.0632	0.0510	0.1293	0.0632	0.0510
	400	0.0822	0.0374	0.0123	0.0374	0.0123	0.0158
	1600	0.1158	0.0333	0.0027	0.0027	0.0002	0.0015
	6400	0.1984	0.1142	0.0523	0.0344	0.0757	0.9007
LTS	25		0.0961	0.5767			0.0961
	100	0.0610	0.0650	0.0708	0.0610	0.0650	0.0708
	400	0.0517	0.0536	0.0611	0.0536	0.0611	0.1177
	1600	0.0523	0.0558	0.0543	0.0543	0.0600	0.1891
	6400	0.0506	0.0525	0.0516	0.0524	0.0577	0.4123
SLTS	25		0.2149	0.6942			0.2149
	100	0.0924	0.1254	0.1797	0.0924	0.1254	0.1797
	400	0.0690	0.0891	0.1256	0.0891	0.1256	0.2147
	1600	0.0639	0.0762	0.0890	0.0890	0.1223	0.3177
	6400	0.0578	0.0639	0.0695	0.0843	0.1208	0.5473

In DGPs 7-12, the $T - h$ outliers are equally distributed in G outlier episodes. The episodes are equally spaced through the sample. In this table $\alpha = -1$

Table 17: Simulated rejection frequencies for nominal 5% tests on ν.

method	n	DGP7	DGP8	DGP9	DGP10	DGP11	DGP12
OLS	25		0.3946	0.3219			0.3946
	100	0.2760	0.1094	0.0693	0.2760	0.1094	0.0693
	400	0.0685	0.0197	0.0132	0.0197	0.0132	0.0344
	1600	0.0096	0.0020	0.0006	0.0006	0.0010	0.0059
	6400	0.0024	0.0000	0.0000	0.0000	0.0000	0.0000
LTS	25		0.1135	0.6079			0.1135
	100	0.0723	0.0674	0.0748	0.0723	0.0674	0.0748
	400	0.0543	0.0537	0.0613	0.0537	0.0613	0.0663
	1600	0.0536	0.0562	0.0552	0.0552	0.0408	0.3015
	6400	0.0499	0.0532	0.0516	0.0220	0.1273	0.3988
SLTS	25		0.2136	0.7357			0.2136
	100	0.1050	0.1323	0.2026	0.1050	0.1323	0.2026
	400	0.0728	0.0881	0.1241	0.0881	0.1241	0.1945
	1600	0.0651	0.0755	0.0880	0.0880	0.0945	0.4539
	6400	0.0575	0.0621	0.0693	0.0442	0.2106	0.5322

In DGPs 7-12, the $T - h$ outliers are equally distributed in G outlier episodes. The episodes are equally spaced through the sample. In this table $\alpha = -0.2$

Table 18: Simulated rejection frequencies for nominal 5% tests on ω.

method	n	DGP13	DGP14	DGP15	DGP16	DGP17	DGP18
OLS	25	1.0000	1.0000		0.8375	1.0000	1.0000
	100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
LTS	25	0.0635	0.0699		0.0665	0.0635	0.0699
	100	0.0501	0.0526	0.0508	0.0550	0.0526	0.0508
	400	0.0533	0.0528	0.0544	0.0508	0.0544	0.0495
	1600	0.0526	0.0464	0.0492	0.0483	0.0533	0.0505
	6400	0.0474	0.0495	0.0497	0.0443	0.0498	0.0496
SLTS	25	0.1492	0.2167		0.0998	0.1492	0.2167
	100	0.0887	0.1243	0.1946	0.0870	0.1243	0.1946
	400	0.0738	0.0923	0.1286	0.0819	0.1286	0.1964
	1600	0.0615	0.0685	0.0856	0.0801	0.1247	0.1960
	6400	0.0531	0.0610	0.0698	0.0726	0.1205	0.1967

In DGPs 13-18, the $T - h$ outliers are located as single outliers and separated in equal spaces. In this table $\alpha = -1$

Table 19: Simulated rejection frequencies for nominal 5% tests on ω.

method	n	DGP13	DGP14	DGP15	DGP16	DGP17	DGP18
OLS	25	0.9998	1.0000		0.8355	0.9998	1.0000
	100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1600	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	6400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
LTS	25	0.0630	0.0722		0.0748	0.0630	0.0722
	100	0.0503	0.0545	0.0500	0.0544	0.0545	0.0500
	400	0.0525	0.0533	0.0549	0.0506	0.0549	0.0488
	1600	0.0523	0.0463	0.0488	0.0488	0.0534	0.0516
	6400	0.0478	0.0494	0.0494	0.0436	0.0505	0.0498
SLTS	25	0.1517	0.2159		0.1106	0.1517	0.2159
	100	0.0897	0.1241	0.1953	0.0860	0.1241	0.1953
	400	0.0737	0.0907	0.1281	0.0817	0.1281	0.1961
	1600	0.0615	0.0689	0.0859	0.0803	0.1247	0.1960
	6400	0.0535	0.0614	0.0700	0.0725	0.1204	0.1972

In DGPs 13-18, the $T - h$ outliers are located as single outliers and separated in equal spaces. In this table $\alpha = -0.2$

Table 20: Simulated rejection frequencies for nominal 5% tests on α.

method	n	DGP13	DGP14	DGP15	DGP16	DGP17	DGP18
OLS	25	0.0543	0.0838		0.0596	0.0543	0.0838
	100	0.0211	0.0619	0.2301	0.0173	0.0619	0.2301
	400	0.0204	0.0528	0.2036	0.0341	0.2036	0.6802
	1600	0.0239	0.0457	0.2007	0.1231	0.6838	0.9951
	6400	0.0346	0.0482	0.1943	0.5262	0.9982	1.0000
LTS	25	0.0669	0.0704		0.0601	0.0669	0.0704
	100	0.0555	0.0550	0.0553	0.0508	0.0550	0.0553
	400	0.0487	0.0520	0.0454	0.0538	0.0454	0.0547
	1600	0.0502	0.0501	0.0500	0.0510	0.0476	0.0490
	6400	0.0456	0.0475	0.0466	0.0516	0.0471	0.0531
SLTS	25	0.1569	0.2180		0.0916	0.1569	0.2180
	100	0.0921	0.1249	0.1979	0.0802	0.1249	0.1979
	400	0.0692	0.0901	0.1169	0.0846	0.1169	0.2001
	1600	0.0608	0.0702	0.0861	0.0834	0.1155	0.1902
	6400	0.0525	0.0578	0.0667	0.0808	0.1205	0.2040

In DGPs 13-18, the $T - h$ outliers are located as single outliers and separated in equal spaces. In this table $\alpha = -1$

Table 21: Simulated rejection frequencies for nominal 5% tests on α.

method	n	DGP13	DGP14	DGP15	DGP16	DGP17	DGP18
OLS	25	0.3216	0.4474		0.1339	0.3216	0.4474
	100	0.1827	0.6381	0.8677	0.1120	0.6381	0.8677
	400	0.1404	0.7136	0.9972	0.4658	0.9972	1.0000
	1600	0.1065	0.5906	0.9999	0.9879	1.0000	1.0000
	6400	0.0833	0.4205	0.9983	1.0000	1.0000	1.0000
LTS	25	0.0739	0.0831		0.1794	0.0739	0.0831
	100	0.0595	0.0566	0.0568	0.0627	0.0566	0.0568
	400	0.0549	0.0500	0.0502	0.0537	0.0502	0.0540
	1600	0.0547	0.0510	0.0497	0.0512	0.0509	0.0540
	6400	0.0479	0.0511	0.0461	0.0496	0.0485	0.0525
SLTS	25	0.1680	0.2308		0.2372	0.1680	0.2308
	100	0.1018	0.1285	0.2042	0.0975	0.1285	0.2042
	400	0.0742	0.0884	0.1167	0.0874	0.1167	0.2022
	1600	0.0650	0.0729	0.0924	0.0825	0.1243	0.1987
	6400	0.0552	0.0622	0.0669	0.0842	0.1238	0.1945

In DGPs 13-18, the $T - h$ outliers are located as single outliers and separated in equal spaces. In this table $\alpha = -0.2$

Table 22: Simulated rejection frequencies for nominal 5% tests on κ.

method	n	DGP13	DGP14	DGP15	DGP16	DGP17	DGP18
OLS	25	0.0715	0.0514		0.0457	0.0715	0.0514
	100	0.0239	0.0246	0.0312	0.0242	0.0246	0.0312
	400	0.0077	0.0114	0.0203	0.0097	0.0203	0.0330
	1600	0.0062	0.0070	0.0112	0.0096	0.0215	0.0341
	6400	0.0063	0.0050	0.0058	0.0124	0.0243	0.0316
LTS	25	0.0710	0.0682		0.0810	0.0710	0.0682
	100	0.0548	0.0531	0.0528	0.0542	0.0531	0.0528
	400	0.0487	0.0516	0.0531	0.0504	0.0531	0.0476
	1600	0.0474	0.0508	0.0509	0.0509	0.0507	0.0511
	6400	0.0508	0.0491	0.0505	0.0499	0.0487	0.0485
SLTS	25	0.1619	0.2143		0.1150	0.1619	0.2143
	100	0.0929	0.1281	0.1994	0.0863	0.1281	0.1994
	400	0.0688	0.0891	0.1220	0.0793	0.1220	0.1956
	1600	0.0558	0.0694	0.0883	0.0833	0.1252	0.1981
	6400	0.0573	0.0617	0.0716	0.0799	0.1201	0.1985

In DGPs 13-18, the $T - h$ outliers are located as single outliers and separated in equal spaces. In this table $\alpha = -1$

Table 23: Simulated rejection frequencies for nominal 5% tests on κ.

method	n	DGP13	DGP14	DGP15	DGP16	DGP17	DGP18
OLS	25	0.3981	0.4824		0.0792	0.3981	0.4824
	100	0.0939	0.1518	0.2455	0.0747	0.1518	0.2455
	400	0.0226	0.0420	0.0791	0.0310	0.0791	0.1564
	1600	0.0098	0.0154	0.0327	0.0248	0.0652	0.1180
	6400	0.0084	0.0073	0.0126	0.0255	0.0631	0.1054
LTS	25	0.1042	0.1029		0.1872	0.1042	0.1029
	100	0.0665	0.0611	0.0608	0.0660	0.0611	0.0608
	400	0.0533	0.0534	0.0557	0.0531	0.0557	0.0500
	1600	0.0494	0.0517	0.0505	0.0522	0.0521	0.0514
	6400	0.0506	0.0499	0.0509	0.0501	0.0490	0.0486
SLTS	25	0.1766	0.2218		0.2236	0.1766	0.2218
	100	0.1063	0.1384	0.2082	0.0996	0.1384	0.2082
	400	0.0724	0.0916	0.1246	0.0848	0.1246	0.1958
	1600	0.0592	0.0706	0.0898	0.0844	0.1256	0.1998
	6400	0.0566	0.0617	0.0709	0.0796	0.1205	0.1984

In DGPs 13-18, the $T - h$ outliers are located as single outliers and separated in equal spaces. In this table $\alpha = -0.2$

Table 24: Simulated rejection frequencies for nominal 5% tests on ν.

method	n	DGP13	DGP14	DGP15	DGP16	DGP17	DGP18
OLS	25	0.1149	0.1242		0.0322	0.1149	0.1242
	100	0.1308	0.1565	0.3048	0.1088	0.1565	0.3048
	400	0.2207	0.3138	0.5265	0.2629	0.5265	0.8218
	1600	0.3820	0.6303	0.8861	0.8275	0.9853	0.9992
	6400	0.6180	0.9297	0.9992	1.0000	1.0000	1.0000
LTS	25	0.0795	0.1003		0.0884	0.0795	0.1003
	100	0.0601	0.0649	0.0739	0.0563	0.0649	0.0739
	400	0.0510	0.0555	0.0604	0.0543	0.0604	0.0765
	1600	0.0509	0.0521	0.0581	0.0570	0.0628	0.0720
	6400	0.0497	0.0470	0.0547	0.0541	0.0599	0.0720
SLTS	25	0.1495	0.2180		0.1163	0.1495	0.2180
	100	0.0944	0.1286	0.1950	0.0831	0.1286	0.1950
	400	0.0677	0.0878	0.1223	0.0827	0.1223	0.1980
	1600	0.0601	0.0710	0.0909	0.0853	0.1233	0.1937
	6400	0.0568	0.0561	0.0731	0.0819	0.1228	0.1920

In DGPs 13-18, the $T - h$ outliers are located as single outliers and separated in equal spaces. In this table $\alpha = -1$

Table 25: Simulated rejection frequencies for nominal 5% tests on ν.

method	n	DGP13	DGP14	DGP15	DGP16	DGP17	DGP18
OLS	25	0.3350	0.3971		0.0698	0.3350	0.3971
	100	0.2730	0.5165	0.9269	0.2220	0.5165	0.9269
	400	0.3097	0.6004	0.9833	0.4618	0.9833	1.0000
	1600	0.4352	0.7592	0.9937	0.9625	1.0000	1.0000
	6400	0.6363	0.9484	0.9999	1.0000	1.0000	1.0000
LTS	25	0.0981	0.1137		0.2165	0.0981	0.1137
	100	0.0705	0.0732	0.0881	0.0712	0.0732	0.0881
	400	0.0537	0.0591	0.0673	0.0574	0.0673	0.0826
	1600	0.0536	0.0557	0.0587	0.0559	0.0652	0.0787
	6400	0.0505	0.0465	0.0550	0.0577	0.0585	0.0725
SLTS	25	0.1654	0.2159		0.2475	0.1654	0.2159
	100	0.1021	0.1357	0.2009	0.0999	0.1357	0.2009
	400	0.0730	0.0894	0.1270	0.0824	0.1270	0.1994
	1600	0.0632	0.0714	0.0896	0.0836	0.1257	0.1995
	6400	0.0575	0.0584	0.0735	0.0845	0.1191	0.1950

In DGPs 13-18, the $T - h$ outliers are located as single outliers and separated in equal spaces. In this table $\alpha = -0.2$

References

Berenguer-Rico, V. and Nielsen, B. (2024). Least trimmed squares: Nuisance parameter free asymptotics. *Econometric Theory*. To appear.